Acalculia from a right hemisphere lesion
Dealing with “where” in multiplication procedures

Alessia Granà, Riccardo Hofer, Carlo Semenza

Dipartimento di Psicologia, Università degli Studi di Trieste, via S. Anastasio 12, 34124 Trieste, Italy
Istituto di Medicina Fisica e Riabilitazione, via Gervasutta 48, 33100 Udine, Italy

Received 26 September 2005; received in revised form 12 June 2006; accepted 18 June 2006
Available online 17 August 2006

Abstract

The present study describes in detail, for the first time, a case of failure with multiplication procedures in a right hemisphere damaged patient (PN). A careful, step-by-step, error analysis made possible to show that an important portion of PN’s errors could be better explained as spatial in nature and specifically related to the demands of a multi-digit multiplication. These errors can be distinguished from other types of errors, including those, expected after a right hemisphere lesion, determined by a generic inability to deal with spatial material, or from other deficits, like neglect, affecting cognitive capacities across the board.

The best explanation for PN’s problems is that he might have difficulties in relying on a visuo-spatial store containing a layout representation specific to multiplication. As a consequence, while knowing what, when and how to carry out the various steps, PN does not know where. What he may thus lack is a spatial schema of multiplication. Such schema is thought to help normal calculators in overcoming working memory demands of complex calculation by representing the information of where exactly each sub-step should be placed.

Keywords: Spatial layout for multiplication; Multi-digit written calculation; Partial products in multiplication; Factor selection in multiplication; Spatial acalculia

1. Introduction

The present study describes in detail, for the first time, a case of failure with multiplication procedures in a right hemisphere damaged patient. In this context, problems will be addressed arising with the current distinction between primary and secondary acalculia and with the definition of spatial acalculia. An effort will be made at starting to identify which and to what extent sub-components of the calculation process may need the support of spatial cognition.

1.1. The nature of deficits in arithmetical procedures and their localisation

The pattern of impairment described in single case studies of deficit in arithmetical procedures used in multi-digit calculation varies from case to case. In a first effort, McCloskey, Caramazza, and Basili (1985) had provided a few examples of a variety of errors with procedures, such as problems in carrying and borrowing with misordering of steps, misalignment, failure of organising intermediate products, failure in integrating intermediate products, etc., in four patients whose full symptomatology, however, was not reported.

More detailed case studies appear in the literature of the past decade. Lucchelli and De Renzi (1993) described what were mostly errors on carrying procedures in a patient who was relatively more proficient with arithmetical fact retrieval. In this case a vascular lesion involved areas 10, 11, 16, 14, 23, 6 and 4 and anterior half of corpus callosum.

A similar dissociation was reported by Temple (1991) in a case of developmental dyscalculia. In patient MT, described by Girelli and Delazer (1996), who also had problems with retrieving single-digit dyscalculia, In patient MT, described by Girelli and Delazer (1996), who also had problems with retrieving single-digit multiplication (which prevented studying procedures in multi-digit multiplication), a special kind of procedural error (“bug”) was identified in subtraction. This error consisted of the systematic application of a disturbed underlying algorithm: MT subtracted the smaller number from the larger one irrespective of the position. In this case the lesion was provoked by a left fronto-temporo-parietal cerebro-vascular accident.
Recently Sandrini, Miozzo, Cotelli, and Cappa (2003) replicated this finding in a very severe fluent aphasic patient who suffered of a haemorrhage in the left tempo-parietal lobe. Semenza, Miceli, and Girelli (1997) described patient MM, affected by hydrocephalus (with more severe effects in the right upper parietal and right dorso-frontal areas), who was instead perfect with arithmetical facts but made unsystematic and progressively more frequent procedural errors in multi-digit multiplication. MM, who always started in the correct way, had remarkable difficulties in ending the operation, and showed little awareness about the precision of his performance. For these reasons Semenza et al. proposed that MM was affected by a “monitoring deficit”, that is an inability to monitor the sequence of operations that calculation procedures specify rather than reflecting faulty knowledge of the algorithm to be implemented. In contrast, cases like Girelli and Delazer’s (1996), mentioned above, could be characterised as having a memory deficit for a procedural algorithm. Granà, Girelli, and Semenza (2001) reported another case (patient FS) who (following a traumatic brain injury damaging the left tempo-parietal areas and a small area in the fronto-parietal right hemisphere), again in presence of intact arithmetical facts knowledge, showed a selective procedural deficit in multi-digit operations. FS never decomposed intermediate products, thus leading to highly implausible results that he judged instead as plausible and correct. Finally, McNeil and Burgess (2002) recently reported a further case of procedural deficit in a probable Alzheimer’s dementia. This case presented similarities with MM, including preservation of arithmetical facts. None of the two cases however seem interpretable, as suggested by McNeil and Burgess, only as a task-switching problem. At least in MM’s case most errors were found when switching was not required. Furthermore, task switching differentiates multiplication from addition only in the final step of the operation, i.e., when, after calculating partial products, these must be added: if switching tasks were the problem, one would expect more errors at this point, which, at least in MM’s case, did not happen.

This brief perusal reveals that described cases derive from either left sided or bilateral/diffused lesions. No single case description of procedure deficit is reported after damage confined to the right hemisphere.

1.2. Calculation in the right hemisphere

Acalculia following right hemisphere lesions has been consistently found in group studies, though in various proportions, probably due to the different sensitivity of different calculation tests (e.g., Ardila & Rosselli, 1994; Basso, Burgio, & Caporali, 2000; Dahmen, Hartje, Bussing, & Sturm, 1982; Grafman, Passafiume, Faglioni, & Boller, 1982; Hécaen, 1962; Hécaen & Angelergues, 1961). Hécaen, Angelergues, and Houiller (1961) classified 183 acalculic patients with retrorolandic lesions according to the clinical signs of: (a) difficulties in the reading and writing numerals: *alexia or agraphia for number* (34.5% of the cases); (b) difficulties in the execution of arithmetical operations: *anarithmetia* (39.3% of the cases); (c) difficulties in the spatial arrangement of numbers: *spatial acalculia*. This latter variety was assumed to be present in 26.2% and was essentially considered as secondary to visuo-spatial troubles (see below). The calculation disorder of spatial type was present in 23.5% of patients with right retrorolandic lesions, in 17% of patients with bilateral lesions and only in 2% of patients with left retrorolandic lesions.

Grafman et al. (1982) also found that right hemisphere damaged patients were impaired in written multi-digit calculation, but the type of errors were not analysed. Dahmen et al. (1982) compared the performance of Broca’s aphasics, Wernicke’s aphasics and right brain damaged subjects in a variety of different tasks, tapping what they termed verbal and visuo-spatial components of arithmetic tasks, not including multi-digit calculation. Their main conclusion was that, since both Wernicke’s aphasics and right hemisphere patients showed spatial disorders, spatial disorders in Wernicke’s aphasia could not be secondary to language impairment. Ardila and Rosselli (1994) tested 21 patients with right hemisphere damage (6 pre-Rolandic and 15 retro-Rolandic) with a “special calculation battery”. Their findings suggest that acalculia appeared particularly in written calculation and would be better preserved in mental calculation. In reading and writing numbers they found spatial alexia and agraphia, leading to particular errors (feature and digit addition, inability to use the spaces to join and separate numbers, difficulty in conserving the written line in a horizontal position, increased left margins and unsteadiness in maintaining left margins, disrespect of spaces and spatial disorganisation of written material). As far as the calculation system was concerned, they found loss of calculation automatisms and reasoning errors (impossible results were not rejected). It can be doubted that these errors are specific to right brain damage. Ardila and Rosselli (1994) investigation was too limited to draw less than generic conclusions (e.g., written calculation was evaluated on only four items), but can give at least some indications about what kind of errors may appear in right hemisphere damaged subjects. Basso et al. (2000) analysed the deficit in number processing in 50 left and 26 right brain damaged subjects. Patients were classified as acalculic if they failed in more than three tasks on the EC301 Battery for Calculation and Number Processing. Following these criteria, and excluding patients whose symptoms could be affected by unilateral neglect, Basso et al. diagnosed as acalculic only 3 out of 26 right brain damaged patients. However, the exact data of these three patients on written calculation were not reported. Single cases of acalculia in right brain damaged patients are indeed rare and rather anecdotal (e.g., Ardila & Rosselli, 2002; Leleux, Kaiser, & Lebrun, 1979).

Bilateral activation, at least of the parietal lobes, is the most common finding in neuroimaging studies on calculation (Chocho, Cohen, van de Moortele, & Dehaene, 1999; Cohen, Dehaene, Chocho, Lehericy, & Naccache, 2000; Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Eger, Sterzer, Russ, Giraud, & Kleinschmidt, 2003; Hubbard, Piazza, Pinel, & Dehaene, 2005; Menon, Rivera, White, Glover, & Reiss, 2000; Naccache & Dehaene, 2001; Piazza, Mechelli, Buttersworth, & Price, 2002; Pinel, Dehaene, Riviere, & LeBihan, 2001; Rickard et al., 2000; Roland & Friberg, 1985; Thioux, Pesenti, De Volder, & Seron 2002; Zago
Evidence of relatively more intense activation on the right seems to be possibly related to proficiency (Zago et al., 2001). More specifically, however, and of relevance to the interests of the present study, no investigation so far has been made on complex written multiplication. It may be nonetheless useful to mention in some detail the main results of an accurate review and meta-analysis of recent imaging and patient studies on calculation recently provided by Dehaene et al. (2003); see also Dehaene, Molko, Cohen, & Wilson (2004). The authors’ conclusion is that the intraparietal, angular and posterior parietal regions play distinct functional roles in arithmetic. In particular, the horizontal segment of the bilateral intraparietal sulcus is systematically activated whenever numbers are processed, independently of the notation used for numbers, and the activation increases when the task requires the processing of quantities. This area is activated in calculation (Burbaud et al., 1999; Chochon et al., 1999; Pesenti, Thioux, Seron, & De Volder, 2000; Rickard et al., 2000), more in approximate than in exact calculation (Dehaene et al., 1999) and more in subtraction than in multiplication (Chochon et al., 1999; Lee, 2000). The other two circuits involved in number processing, the left angular verbal system and the posterior superior parietal attention system, rely on representations and processes that are not specific to the number domain. Particularly, the left angular gyrus supports verbal aspects of numerical processing (Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002; Stanescu-Cosson et al., 2000). It mediates the retrieval of facts stored in verbal memory, but not other numerical tasks, like subtraction, number comparison or complex calculation, related to processing of quantity (Chochon et al., 1999; Delazer, Domahs et al., 2003; Duffau et al., 2002; Lee, 2000). The posterior superior parietal lobule, instead, supports visuo-spatial processes, attention and spatial working memory (Corbetta, Kincade, Ollinger, McAvoy, & Shulman, 2000; Culham & Kanwisher, 2001; Pesenti et al., 2000; Pinel et al., 2001; Simon et al., 2002) related to numerical processing. It is activated in number comparison (Pesenti et al., 2000; Pinel et al., 2001), approximation (Dehaene et al., 1999) or counting (Piazza et al., 2002; Piazza, Giacomini, Le Bihan, & Dehaene, 2003). It also appears to increase in activation when subjects carry out two operations instead of one (Menon et al., 2000). In conclusion, the role of different structures within the neural network supporting complex mental calculation has been widely investigated but little is still known about the neural implementation of calculation when further increment in complexity requires resorting to the written modality. However, in order to collect this information in a sensible way, and before looking for precise anatomical locations, it is important to preliminarily disentangle conceptually each component of calculation at this more complex level. To this aim the study of dissociations offered by neuropsychological cases may indeed still be a fundamental tool.

1.3. The contribution of spatial cognition to calculation: an old neuropsychological problem

The largest part of neuropsychological research on number processing and calculation concerns the verbal component. A strong relationship between language and numbers has been found in numerous studies (e.g., Seron, 2001; Spelke & Tsivkin, 2001). However, evidence for the contrary is also on record. Hermelin and O’Connor (1990) found astounding numerical abilities in a languageless autistic young man and Rossor, Warrington, and Cipolotti (1995) found a severely aphasic patient with left temporal lobe atrophy still being able to calculate (see Giaquinto, 2001). In contrast, the spatial component has been defined rather intuitively, and remains vaguely and implicitly acknowledged (see Caramazza & McCloskey, 1987; Miceli & Capasso, 1999), being, paradoxically, relatively more dealt with, as clearly shown in an excellent review by Hartje (1987), in the old neuropsychological literature than in more recent investigations. Indeed, some of the historical papers (e.g., Peritz, 1918; e.g., Ehrenwald, 1931; Kleist, 1934; Krapf, 1937; Leonhard, 1938; Singer & Low, 1933), based on single cases and phenomenological observation, found a close relationship between space and arithmetical and numerical tasks.

The notion of “spatial acalculia” (SA) is nowadays generally used to refer to calculation deficits that are intuitively considered to be secondary to spatial problems. In their seminal paper, Hécaen et al. (1961) applied the label of “acalculia of spatial type” to a group of patients who were “unable to respect the position and the order of digits in relation to one another”, and extended the notion to include troubles to calculation deriving from spatial neglect and from “spatial inversions of operational mechanisms”. The label spatial acalculia is thus given to patients failing in the visuo-spatial components of arithmetic tasks: a list of these failures appears in Hartje’s review, that also lists the types of spatial disorders that would affect calculation processing. However, this list of putative errors lacks a link to empirical observations and does not clearly distinguish which errors should be considered secondary to visuo-spatial disorders (e.g., visuo-spatial neglect) and which errors could primarily stem from a disorder of operation specific visuo-spatial memory.

For the purpose of the present investigation only cases where a spatial disorder would affect in a visible way arithmetical procedures must be considered. In fact, in all previous studies on failure on arithmetical procedures, the errors in written operations were never classified according to the different operation steps (i.e., in the case of multiplication, factor selection, arithmetical fact retrieval, carry, calculation of partial products, addition of partial products). It thus remains unclear precisely in which steps right hemisphere acalculic subjects err the most and why, and, in particular, to what extent the found deficits are secondary to visuo-spatial functions.

Models introduced by cognitive neuropsychology are so far of little help in the enterprise of disentangling spatial from nonspatial components in complex calculation and of identifying their neural localisation. Even the “triple code model” proposed by Dehaene and Cohen (1995), which is the only one that tries to tie specific cognitive processes to anatomical locations, does not really fill this gap of knowledge. In fact it only implies the basic components of number processing: none of these representations can explain how humans are able to solve multi-digit complex multiplications, how much do they rely on spatial cognition and
where is the algorithm for the different operations (addition, subtraction, division and multiplication) stored in the brain.

One of the few relevant mentions of a specific spatial component in multi-digit calculation may be found in Dehaene and Cohen (1995, p. 102): “A visuo-spatial store may also be used to maintain on-line the spatial layout and digits of an ongoing multi-digit calculation”. On the same line, Hartje (1987) had earlier observed how, in complex calculation, more or less automated spatial patterns of processing interact with logical reasoning: beyond a certain level of difficulty only very sophisticated subjects could possibly perform complex calculation without resorting to these patterns. However, these reasonable assumptions are not accounted for in Dehaene and Cohen’s (1995) actual model and they were not systematically investigated ever since. Furthermore, no theory has ever been offered about the processes involved in “maintaining the spatial layout on-line” and the LTM representation of the spatial layout itself.

In summary, available literature on the sort of calculation disorder that may be considered typical of a right hemisphere lesion is far from being satisfactory. As Miceli and Capasso (1999) argued, “… from the metatheoretical and the methodological standpoint the reported analyses are insufficiently detailed, the criteria on which they are based are intuitive and are not justified by clear hypotheses on the structure of the cognitive system under analysis”. The very concept of spatial acalculia is indeed ambiguously used. It is understood, as reported above, that in most cases, on an intuitive basis, spatial acalculia is considered secondary to other disturbances. No effort has ever been made in identifying exactly which components of multi-digit calculation may rely on spatial cognition and which pattern of performance should be expected in the case of a specific disturbance of these functions. For instance, Hartje (1987) mentions the fact that “in writing down the intermediate products or the number values to be carried, shifting the numbers to the right will cause errors”. Is it possible to find such shift, and its consequences, independently from spatial disorders determining every configuration to shift to the right? In other words: is it possible to distinguish when the patient shifts to the right because he/she has lost the capacity to explore space from the case where the information regarding the positions of the steps is inaccessible to the patient because of a permanent loss or because of a refractory (or access difficulty) state? No answer to problems of this sort has ever been attempted.

1.4. A starting point

A possible starting point may consist in simply acknowledging, like Dehaene and Cohen (1995) did without pursuing this issue any further, that some basic spatial processing components must be inherent to multi-digit written calculation as it is usually performed. As already reported, Dehaene and Cohen, in particular, mention a “spatial layout” that should be maintained on-line. This component must involve a long term memory representation, specifying what needs to be worked upon and, crucially, where. This representation, indeed, must be maintained on-line in order to perform the calculation. However, beyond a certain level of complexity, the actual calculation may exceed working memory capacities, and writing it down in its components according to a conventional procedure, respecting, in particular, a conventional spatial layout, may become necessary to perform the operation. In order to minimise the effort, long term memory must therefore contain not only the information of what is worked upon and how, but also of where the actual figures need to be written. Mastering this information (mentioned in Hartje as “automatised spatial patterns of processing”) ultimately makes complex calculation more automatic, less in need of continuous back-up and therefore more easily accessible to the calculating individual. Indeed, in principle, there is no need to appeal to a spatially encoded, pre-organised schema to facilitate the procedure. It is however hard to believe that the very process of writing down the calculation according to pre-learned conventional order and display may entirely dispense with the help of spatially coded information.

The calculating individual may be, as a result of several causal factors, like brain damage, poor genetic equipment, insufficient training and other unfavourable conditions, unable to lay down the written operation. This may happen either because of an inability to use spatial information properly (e.g., in the instance of disturbances like neglect) or, more specifically, because of a relatively selective disturbance to the stored memory for the proper layout or to its retrieval. These alternatives are, at least in principle, distinguishable. Indeed, it may ultimately be very difficult if possible at all to capture and disentangle precisely all factors determining success in correctly carrying out a complex multi-digit multiplication. However, it does not seem a good approach to pretend, vis-à-vis these difficulties, that information of spatial nature, stored in order to simplify complex multiplication procedures, does not exist.

An answer to the question of whether this analysis reflects psychological reality may come from the observation, carried out with the methods of cognitive neuropsychology, of patients affected by acalculia. The case of a patient who, as a result of right hemisphere damage, has lost the ability to carry on calculation procedures lends itself to such an investigation.

2. Case report

PN was a 32-year-old right-handed businessman with 15 years of formal education. He was pre-morbidly very familiar with simple mathematics, in particular with multi-digit calculation, that he currently used in his business. In December 2002 he suffered from a haemorrhage as a consequence of the rupture of a right middle cerebral artery aneurysm, which was then surgically treated. A CT scan performed in February 2003 showed a vast hemorrhagic intraparenchimal lesion in the right temporoparietal region (see Fig. 1), involving the polar planum, Heschl gyrus, the temporal planum, posterior insula, the supramarginal gyrus and the angular gyrus, with dilatation of the right lateral ventriculum; no lesion in the left hemisphere was detectable. He was left with a very mild left hemiparesis, left hemianopsia and mild cognitive deficits. The investigation on PN’s impairment with procedures in multi-digit calculation was carried out in two different periods, first from February to mid-March 2003, and, second, from the end of March to the end of April 2003.
Before each of these two periods PN underwent a general assessment of cognitive functions, including a routine evaluation of his mathematical skills. He gave his informed consent to the present investigation.

3. Methods

PN’s performance on multi-digit multiplications was compared with that of three age- and education-matched controls in order to better match the quality of their errors with his unusual dealing of multiplication procedures. They never produced procedural errors on the same material administered to PN. The few errors they committed were in fact and/or rule retrieval.

While in educated participants only sporadic errors commonly emerge in multi-digit multiplications, a huge variability in the quantity of errors is found in multi-digit divisions. Division is, indeed, seldom performed manually after school, and most people, having lost familiarity with it, need to take time for retrieving and work out from memory the appropriate procedure. For this reason, in the lack of recent positive evidence of intact performance, division is seldom investigated in neuropsychological work, and was excluded from this study.

Both the first and the second assessment were carried over several sessions. The type of operation and the order of presentation of each operation was balanced over sessions.

4. First assessment

4.1. Neuropsychological background

PN spontaneous speech was fluent, with no aphasic signs in either production or comprehension (Aachen Aphasia Test, Italian version, Luzzati, Willmes, & DeBleser, 1991). Repetition and automatic speech were normal. His oral comprehension of words and sentences assessed was perfect. Both reading and writing were affected by visual neglect. He performed well on a naming task (score = 76; mean score = 64; S.D. = 2; BORB) and on a verbal phonological fluency task (score = 35.8; mean score = 36.28; S.D. = 10.81; Carlesimo, Caltagirone, Gainotti, & Nocentini, 1995).

His digit span forward was well preserved in the auditory modality (span = 6) and his visuo-spatial span was within normal limits, though at the lower level (Corsi span = 4, mean score for young adults = 5.11; S.D. = 1.01; testing material and norms taken from Spinmiller & Tognoni, 1987). The evaluation of his long-term memory showed learning and retrieval diffi-
His performance on the Rey auditory-verbal learning test was within the norms for immediate recall (score = 34.3; mean score = 50.12; S.D. = 8.35; Carlesimo et al., 1995) but poor for delayed recall (score = 4.2; mean score = 11.37; S.D. = 2.17; Carlesimo et al., 1995). In the Babcock story recall PN was impaired (score = 9.34; cut-off = 15.76; De Renzi, Faglioni, & Ruggerini, 1987), also showing some difficulties in focusing attention on the material to be stored. In fact, his attentional functions were partly disturbed. Backward digit span was poor (span = 3) as well as selective visual attention (score = 16; mean score for young adults = 53.54; S.D. = 6.76; Spinnler & Tognoni, 1987).

An impaired performance was found in the Albert’s test (score = 21; Albert, 1973), in the Bell test (7 out of 35; Gauthier, Dehaut, & Joanette, 1989) and also the letter cancellation devised by Diller and Weinberg (1977) was greatly affected by unilateral neglect (the difference between the omission on the right and the left side of the sheet was 37; cut-off ≥ 3, Vallar, Rusconi, Fontana, & Musicco, 1994). Moreover, when PN was required to draw from memory a clock he displaced all the hours to the right. Instead, no imaginal unilateral neglect was detected when PN was required to describe the mental image of some familiar place from a given vantage point (e.g., tests equivalent to Bisiach’s & Luzzatti, 1978, “Duomo di Milano” test). PN also did not show any sign of personal neglect detectable with available techniques (Beschin & Robertson, 1997). A mild constructional apraxia was also observed, while no ideomotor or ideational apraxia was detected. Visual discrimination was normal (nine out of nine on Poppelreuter’s test). Average scores were found in telling differences between words and in interpretation of proverbs and criticisms of absurd stories (score = 40.25; mean score for young adults = 51.93; S.D. = 6.22; Spinnler & Tognoni, 1987).

The first neuropsychological assessment included the administration of an experimental battery for number processing and calculation; the results are presented in Table 1.

PN showed a well preserved ability in all transcoding tasks between the Arabic, alphabetic and phonological codes. His comprehension of both Arabic and written verbal numerals up to five-digits was flawless as indicated by his performance in the number comparison task. He was able to recognise the four arithmetical signs and to write down an operation. Although sometimes slow, he demonstrated a preserved knowledge of arithmetical signs and to write down an operation. Although sometimes slow, he demonstrated a preserved knowledge of orally presented arithmetical facts including both fact- and rule-based problems. The former are assumed to be directly retrieved orally presented arithmetical facts including both fact- and rule-based problems. The former are assumed to be directly retrieved based on long-term memory where they are stored in the form of general rules (e.g., \( N \times 1 = N; 0 \times N = 0 \), for any number). A multiple-choice task of multiplication facts where he had to choose the correct answer among alternatives (measured through the calculation battery from Delazer, Girelli, Grana, & Domahs, 2003) was also well performed. Multi-digit calculation for additions, subtractions and multiplications was instead clearly impaired (22% correct). The errors reflected a failure to use the carry and borrow procedure. Interestingly, spatial errors (e.g., misalignment of the numbers in their appropriate columns) for multiplications problems were also observed. Moreover, approximation (measured through the calculation battery from Delazer, Girelli et al., 2003) was impaired: PN was unable to choose between four alternatives the correct answer to a multi-digit calculation task. Mental calculation (measured through the calculation battery from Delazer, Girelli et al., 2003) was instead good for subtractions but clearly impaired for additions, multiplication and divisions. Finally, his conceptual knowledge (measured through the calculation battery from Delazer, Girelli et al., 2003) was severely compromised as evidenced by the arithmetical principles task (e.g., \( 24 + 37 = 61; 37 + 24 = ? \); or \( 12 \times 4 = 48; 12 + 12 + 12 + 12 = ? \)).

In summary, the preliminary assessment of PN’s numerical skills indicated well preserved number processing abilities, together with specific difficulties in calculation. The good performance in arithmetical facts, in contrast with impaired multi-digit written calculation, was the starting-point for further testing.

### Table 1

<table>
<thead>
<tr>
<th>Subtest</th>
<th>First period</th>
<th>Second period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Dots counting</td>
<td>18/20</td>
<td>20/20</td>
</tr>
<tr>
<td>2. Transcoding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Repetition</td>
<td>15/15</td>
<td>15/15</td>
</tr>
<tr>
<td>Written verbal → spoken verbal</td>
<td>15/15</td>
<td>15/15</td>
</tr>
<tr>
<td>Spoken → written verbal</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>Arabic → written verbal</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>Arabic → spoken verbal</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>Written verbal → arabic</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>Spoken verbal → arabic</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>3. Recognition of arithmetical signs</td>
<td>12/12</td>
<td>12/12</td>
</tr>
<tr>
<td>4. Number comparison</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Written verbal numerals</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>Arabic numerals</td>
<td>20/20</td>
<td>20/20</td>
</tr>
<tr>
<td>6. Writing down an operation</td>
<td>8/8</td>
<td>8/8</td>
</tr>
<tr>
<td>7. Arithmetical facts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td>44/44</td>
<td>44/44</td>
</tr>
<tr>
<td>Addition</td>
<td>25/25</td>
<td>25/25</td>
</tr>
<tr>
<td>Subtraction</td>
<td>23/25</td>
<td>25/25</td>
</tr>
<tr>
<td>8. Multiplication multiple choice</td>
<td>34/36</td>
<td>36/36</td>
</tr>
<tr>
<td>9. Written calculation (three addition, three subtraction, three multiplication)</td>
<td>2/9</td>
<td>4/9</td>
</tr>
<tr>
<td>10. Approximate calculation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td>0/3</td>
<td>0/3</td>
</tr>
<tr>
<td>Subtraction</td>
<td>0/3</td>
<td>0/3</td>
</tr>
<tr>
<td>Multiplication</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>Division</td>
<td>0/3</td>
<td>0/3</td>
</tr>
<tr>
<td>11. Mental calculation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>Subtraction</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>Multiplication</td>
<td>1/3</td>
<td>2/3</td>
</tr>
<tr>
<td>Division</td>
<td>0/3</td>
<td>1/3</td>
</tr>
<tr>
<td>12. Principles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additions</td>
<td>4/15</td>
<td>14/15</td>
</tr>
<tr>
<td>Multiplications</td>
<td>7/15</td>
<td>5/15</td>
</tr>
</tbody>
</table>
and investigations, where multiplication problems and related spatial errors were studied more in detail.

4.2. Experimental tasks

4.2.1. Definition and description of the four elementary arithmetical operations

A definition and, subsequently, a description of procedures was required for all basic arithmetical operations.

4.2.2. Multi-digit operations

Eighty-five multi-digit operations (23 additions, 31 subtractions and 31 multiplications) were visually presented to be answered in written modality. Operations varied in complexity in terms of carrying or borrowing requirements. When “0” was included, its relative position within the numerals was balanced across operations (e.g., $NNN \times N0N; N0N \times NNN$, etc.). PN was asked to work out the solution of each operation with no time pressure and was encouraged to comment on the operational steps used.

Addition. PN was presented with 23 multi-digit additions; 16 of them required the carry procedure. The following structures were included: $2 + 2$-digit ($N=2$); $3 + 2$-digit ($N=3$); $3 + 3$-digit ($N=9$); $4 + 2$-digit ($N=2$); $4 + 3$-digit ($N=7$). Operations were presented in random order.

Subtraction. PN was required to solve 31 multi-digit subtractions; 20 of them required the borrowing procedure. The following structures were included: $2–2$-digit ($N=9$); $3–2$-digit ($N=13$); $3–3$-digit ($N=9$). Operations were presented in random order.

Multiplication. PN was required to solve 31 multi-digit multiplications; 22 of them required the carrying procedure. The following structures were included: $2 \times 1$-digit ($N=1$); $2 \times 2$-digit ($N=4$); $3 \times 1$-digit ($N=8$); $3 \times 2$-digit ($N=10$); $3 \times 3$-digit ($N=6$); $4 \times 2$-digit ($N=2$). Operations were presented in random order.

4.3. Results

4.3.1. Definition and description of the four elementary arithmetical operations

Addition. “Is the numerical sum of a number of digits, for example $6 + 6$ equals 12 or 60 + 60 equals 120”. “One has to take a number and add it; for example, if you were to explain a child how much 2 + 2 makes, take two fingers and give him two other fingers and then you tell him to count how many fingers are there.

Subtraction. “Is a mathematical calculation which foresees to take away numbers from other numbers”. “One needs it when one wants to take away numbers from other number; for example 12 – 12 makes 0, there is no rest, 12 – 1 makes 11, etc.”.

Multiplication. “One has to take numbers and multiply them by other numbers”. “One needs to take some numbers and multiply them by other numbers. It is quicker than an addition, because there is an exponential adding. For example $4 \times 2$ is 4 + 4 and $4 \times 4$ is $4 + 4 + 4 + 4$. It is a quicker calculation”.

Division. “This is the inverse of a multiplication; one has to take numbers and divide them by other numbers”. “If we divide, it is the opposite of a multiplication; for example, 16 is divided by 4 and makes 4 because if I take 4 and multiply it by 4 I get the [same] result”.

4.3.2. Multi-digit operations

Sometimes PN produced more than one error on a single operation; thus the number of errors was larger than the number of operations wrongly produced. The results are presented in Table 2. The three controls participants committed a total of four arithmetical fact retrieval errors and one rule retrieval error.

Addition. PN’s performance was good: he correctly solved 19 operations (82.6% correct) and thus produced only four errors: two missed carry and two omissions of digits.

Subtraction. PN solved correctly 18 operations (58% correct). He made 17 errors: 9 missed borrow, 4 arithmetic facts mistakes, 3 counting errors and 1 digit omission.

Multiplication. PN was severely impaired: all multiplication operations were wrongly produced (0% correct). He made 216 errors that were analysed and classified according to the step in which they occurred (i.e., factor selection; computation of the partial products; computation of the final result) and to the sub-procedures neglected or inappropriately applied (e.g., carry, spatial alignment; retrieval of arithmetic facts).

The following types of errors were observed in the successive steps:

(a) Factor selection: The numbers to be multiplied always corresponded to the correct pair: however PN produced 32 errors (14.8%): 25% of them ($N=8$) were omissions of a digit on the left side and 75% of them ($N=24$) consisted of selection and multiplication of a factor “0” not existing in the actual multiplication. Thus, he performed an extra step that is not required by the multiplication algorithm: writing unnecessary zeros was in this case considered an error. An error of this sort does not lead, of course, to a wrong final result. If, as in the examples reported in Fig. 2a and b, the final result was wrong, such outcome was due to other types of errors. Note that the patient shows a perfect knowledge of zero-rules ($N \times 0 = 0$, etc.). In three cases PN multiplied one digit of the multiplicator with an inexistent “0” at the leftmost side of the multiplicand (Fig. 2a) while in 12 cases he multiplied an inexistent “0” at the leftmost side of the multiplicator with one digit of the multiplicand (Fig. 2a). He always performed this extra step when all digits of the multiplicand were already multiplied. He then wrote the obtained result, “0”, in the partial products. In nine cases, instead, PN selected an inexistent factor “0” on the leftmost side of the multiplicator and multiplied it with another inexistent factor “0” on the leftmost side of multiplicand (Fig. 2b). Again he wrote the obtained result “0” in the partial products. He always verbalised all procedural steps. Note the PN always observes the prescription of proceeding from the right digit leftwards, a part of the procedure that multiplication has
Table 2
Number of correct responses and of errors in solving multi-digit written operations. PN’s distribution of error types in multiplication was divided according to the step in which the errors occurred and to the sub-procedures neglected or inappropriately applied (e.g., carry, spatial alignment; retrieval of arithmetic facts) taking into account the larger first and the smaller first condition in the first and in the second assessment.

<table>
<thead>
<tr>
<th>Operations</th>
<th>Type of errors</th>
<th>FIRST ASSESSMENT</th>
<th>SECOND ASSESSMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>correct/ responses total</td>
<td>partial 1</td>
</tr>
<tr>
<td>Addition</td>
<td></td>
<td>19/23 (82.6%)</td>
<td></td>
</tr>
<tr>
<td>Subtraction</td>
<td></td>
<td>18/31 (58%)</td>
<td></td>
</tr>
<tr>
<td>Larger first multiplication</td>
<td></td>
<td>0/31 (0%)</td>
<td>216</td>
</tr>
<tr>
<td>Calculation of partial products</td>
<td></td>
<td>0/31 (0%)</td>
<td>216</td>
</tr>
<tr>
<td></td>
<td>Factor selection</td>
<td>e.g. fig. 2a</td>
<td>32/216</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. fig. 2b</td>
<td>24/32 (75%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. fig. 2c</td>
<td>9/24 (37.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. fig. 3a</td>
<td>8/32 (25%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. fig. 3b</td>
<td>28/78 (35.9%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. fig. 3c</td>
<td>13/78 (16.7%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. error similar to fig 3d</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calculation of partial products</td>
<td>e.g. error similar to fig 3d</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. error similar to fig 3d</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. error similar to fig 3d</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. error similar to fig 4c</td>
<td></td>
</tr>
<tr>
<td>Smaller first multiplication</td>
<td></td>
<td>70/216 (32.4%)</td>
<td></td>
</tr>
<tr>
<td>Calculation of partial products</td>
<td></td>
<td>70/216 (32.4%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor selection</td>
<td>e.g. fig. 4a</td>
<td>11/70 (15.7%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. fig. 4b</td>
<td>59/70 (84.3%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. error similar to fig 4c</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. error similar to fig 4c</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 2. Examples of PN’s errors in the factor selection (note that other errors not involving factor selection are also present): (a) multiplication of one digit in the multiplicator with a non-existent factor zero at the leftmost side of the multiplicand (e.g. he said: "0 × 4 is 0, write 0; 0 × 6 is 0, write 0; 0 × nothing is 0, write 0") and multiplication of a non-existent factor zero at the leftmost side of multiplicator with one digit in the multiplicand (e.g., he said: “nothing × 4, is 0, write 0; nothing × 6, is 0, write 0; nothing × 4 (that is the carry of 8 × 6) is 0, write 0; nothing × 3 (that is the carry of 8 × 4), is 0, write 0”); (b) multiplication of a non-existent factor zero in the multiplicator with a non-existent factor zero in the multiplicand (e.g., he said “6 × 0, is 0; 6 × 0 is 0; 6 × 5 is 30, write 0 and carry 3 (that is write near the 0), nothing × nothing is 0, write 0”); (c) selection and multiplication of a factor zero that does not exist in the actual multiplication and is not written in the partial product (e.g., he said: “2 × 6 is 12, write 2 and carry 1; 2 × 0 is 0 + 1 (carry) is 1”); (d) multiplication of one digit in the multiplicator with a non-existent factor zero (e.g., he said: “3 × 3 is 9, write 9; 3 × 1 is 3, write 3; 3 × nothing, is 0, write 0; 1 × 3 is 3, write 3; 1 × 1 is 1, write 1; 1 × nothing, is 0, write 0; 1 × 3 is 3, write 3; 1 × 1 is 1, write 1; 1 × nothing, is 0, write 0”).

(b) Calculation of partial products: Once factors to be multiplied were correctly selected, errors occurred in the calculation of partial products (114/216, 52.8% of the total errors). Seventy-eight errors (68.4%) could be attributed to inappropriate use of carry. PN always wrote down the carry (a strategy children are instructed to use in school) but never added it to the next partial product. In 28 cases he reported the carry on the leftmost side of the multiplicand (Fig. 3a) while in 37 cases he wrote it next to partial product (which is how the instruction goes; Fig. 3b). Finally, in 16.7% of cases (N = 13) he decomposed inappropriately the last retrieved arithmetical fact in the row (i.e., the only one that does not need decomposition) and wrote the carry near to the partial product (Fig. 3c). Other errors consisted of omissions of the auxiliary line (always when it was required) resulting in failure in the spatial alignment of the spatial products (22%; N = 25) or in failure of arithmetical fact retrieval (2.6%; N = 3). Few errors (7% error; N = 8) were hard to interpret and were classified as “other”.

(c) Sum of partial products: PN produced 70/216 errors (32.4%). Such errors resulted from spatial misalignment. In this case the error was determined by the inclusion of the carry written near to partial product (N = 11; see Fig. 4a) or by further addition of carries mixed to digits in partial products (N = 59; see Fig. 4b).

5. Second assessment

5.1. Neuropsychological background

Critical tests were repeated in this second assessment, with the addition of other tests aimed at deepening the analysis of the
At this time PN was less impaired, but, while he was perfectly able to perform multi-digit additions and subtractions, he kept committing a considerable amount of errors on multiplications. The results are presented in Table 2. Control participants committed eight fact retrieval errors, three on multiplications with larger number in the multiplicand and five on multiplications with larger number in the multiplicator.

**Addition.** PN’s performance was error free.

**Subtraction.** PN scored fast and flawless.

**Multiplication.** Again he produced more than one error on a single operation: thus the number of errors was larger than the number of operations wrongly produced. Overall, PN was correct in only 58.8% of the cases. He mistook 15 multiplications in the “larger first” arrangement (69.4% correct) and 27 multiplications in “smaller first” arrangement (49% correct). He produced a total of 197 errors: 40 of them in “larger first” and 157 on “smaller first” operations.

Errors were separately counted for the two kinds of arrangement and analysed according to the classification used for the first set of tasks; thus the following types were observed:

1. “Larger first” multiplication:
   (a) Factor selection: PN made nine errors resulting from omissions of the leftmost digits \((N=3)\) or selection and multiplication of a factor “0” not existing in the actual multiplication \((N=6)\). This last extra step, not required by the multiplication algorithm, was performed multiplying one digit of the multiplicator with an inexistent factor “0” of the multiplicand but without writing the result. The

---

**Fig. 4.** Examples of PN’s errors in the sum of partial products (note that other errors not involving the sum of partial products are also present): (a) inclusion of the carry written near to the partial product; (b) addition of carries mixed to digits in partial products; (c) spatial misalignment.
obtained result performed when PN decomposed the last retrieved arithmetical fact (when not appropriate) was not written but verbalised and occurred only once all digits of the multiplicand were already multiplied (Fig. 2c).

(b) Calculation of partial products: PN produced 30 errors in this step. 12/30 errors could result from inappropriate use of carry. While in one case he put down the carry near to the partial product without adding it to the next partial product, in 11 cases he decomposed inappropriately the last retrieved arithmetical fact of each row: in 4 of these cases he added the carry to the next partial product in the row just below, while in 7 cases he put down the carry near to the partial product. Other errors consisted in omissions of the auxiliary line resulting in failure in the spatial alignment of the spatial products (N = 4); in failure of arithmetical fact retrieval (N = 3) or in counting errors (N = 5) when he added the carry to a retrieved arithmetical fact. Six errors were difficult to interpret and were classified as “other”.

(c) Sum of partial products: PN produced only one error resulting from spatial misalignment involving a carry written near to the partial product and the same partial product.

- “Smaller first” multiplication:
  (a) Factor selection: PN produced 49/157 errors: two were omissions of a digit on the left side while 47 resulted, as in the “larger first” arrangement, from selection and multiplication of factor “0” not existing in the actual multiplication. As argued above, this last extra step is not required by the multiplication algorithm and was performed multiplying one digit of the multiplicator with an inexistent factor “0” of the multiplicand. In 37 cases the obtained result “0” was written in the partial products (Fig. 2d), while in 10 cases it was not written but verbalised. This extra step occurred only once all digits of the multiplicand were already multiplied.
  (b) Calculation of partial products: PN produced 97/157 errors. The errors could result from inappropriate use of carry (N = 68) when he decomposed inappropriately the last retrieved arithmetical fact of each row. Particularly, in nine cases he added the carry to the next partial product in the row below (Fig. 3d); while in 59 cases he wrote the carry near to the partial product. Other errors consisted of omissions of the auxiliary line, resulting in failure in the spatial alignment of the spatial products (N = 22; Fig. 3e); in failure of arithmetical fact retrieval (N = 2) or in counting errors (N = 2) when he added the carry to a retrieved arithmetical fact. Three errors were difficult to interpret and were classified as “other”.
  (c) Sum of partial products: PN made 11/157 errors consisting in spatial misalignment between a carry written near to the partial product and the same partial product (Fig. 4c).

In summary his main difficulties in written multiplication (carry procedure, alignment of intermediate products, factor selection) have been amplified when operations where presented in the “smaller-first” arrangement compared to his performance in the “larger-first” arrangement.

6. Discussion

The present paper describes a patient, PN, with an exclusively right sided lesion, who showed, upon administration of a series of mathematical tasks, an interesting, previously unreported, pattern of neuropsychological dissociations in performing multi-digit written multiplication. In contrast with available reports, PN’s study was conducted, longitudinally, on a sizeable sample of operations which allowed to observe very consistent phenomena. PN’s pattern of performance is worth reporting independently of possible interpretations, that per se may be premature: however it will be only through descriptions of this sort and careful comparison among cases that advancement could be expected.

PN showed preservation of arithmetical facts and rule-based arithmetical problems as well as of procedures in addition as well as subtraction (these last only in the second administration): in contrast, his multiplication procedures were severely affected. This notwithstanding, PN was able to describe the procedures in words, and seemed to know the various steps that are necessary to completely perform the operation. However, he committed a number of interesting errors.

Through a careful error analysis, conducted for the first time in a step by step fashion, it is indeed possible to isolate an important portion of PN’s errors that seems to be of a specifically spatial nature, inherent to the demands of a multi-digit multiplication. These errors can be distinguished from other types of errors, including after a right hemisphere lesion, determined by a generic inability to deal with spatial material, or from other deficits, like neglect, affecting cognitive capacities across the board. Indeed it may not be possible to eventually demonstrate that such errors exclusively affect the calculation domain. It is important, however, in absence of a clear proof of the contrary, and vis-à-vis a theory that explains them in calculation-specific spatial terms, to establish that a generic spatial deficit is unlikely to be involved in their origin.

PN’s previously unreported type of errors, not committed by matched controls, may thus be summarised as follows, along with the reasons why they may be considered as spatial in nature or, at least, resulting from the application of strategies aimed at compensating for a spatial difficulty:

(a) Selection and multiplication of a factor “0” that did not exist in the actual multiplication, then adding the carry to the leftmost side of the multiplicand or of the partial products. This type of error is indeed puzzling: however it may be considered as a compensation to a spatial difficulty, insofar the “0” occupied a position at the leftmost border of the factors, when presumably the patient did not know where to proceed. Adding “0” may therefore have been a strategy to keep a border and stop. This may not be the ultimate origin of this strange error. But, indeed, a memorised, well specified spatial layout, clearly deficient in the patient, would
normally help to avoid this error in computing the partial product.

(b) Adding the carry to the leftmost side of the multiplicand or of the partial product. Thus, while the patient knew what the carry was (and if there was any), he seemed to have forgotten the fact that there is not an explicit place for it in the canonical spatial layout of the operation. As compensation he systematically wrote down the carry without adding it to the next partial product. This error could perhaps be explained simply by positing that the patient could not remember what is supposed to be done with the carry digit. The precise origin of such error is hard to understand: however, a full knowledge of the proper spatial layout of the operation would have certainly prevented in this case this particular compensatory strategy and therefore this form of error.

(c) Inappropriate decomposition of the last retrieved arithmetic fact in the row, and carry written near to the leftmost side of the partial product or added to the partial product in the row below. He knew when and how the carry must be added but he seemed to have forgotten where. As compensation he added it nearby, besides or below. The patient seems to ignore that the spatial layout requires independent calculation in each row.

Most of these explanations may, in principle, be objected to: the procedures used by PN may, in fact, be rejected on purely logical grounds. An exhaustive and logically determined control of all necessary steps is however unlikely to be working in multi-digit calculation unless specifically decided. In expert but not theoretically minded calculators, highly automatized processes may instead dispense with such a strict control of logical steps. Even some expert calculators, in fact, would be at loss in spelling out verbally the logic behind the spatial display of the numbers they can quickly, mindlessly but efficiently, lie down on paper in a complex operation. It is therefore simpler to postulate that, normally, human calculators are rather relying on a well established routine. Such routine may be based on an overlearned spatial layout or spatially organised specific procedure. A precise, automatically running, memory for this layout is probably missing in PN, who seemingly resorts to a substitute wrong form of it or to failing strategies to keep the operation going. In other words it is important to stress that, while attentional and other kinds of to space or of space representation are not sufficient performance of spatial disorders of the type known to affect right hemisphere patients, it is possible to contend that such generic disorders of to space or of space representation are not sufficient to produce all the observed types of errors.

In this respect, mind that the evidence that the patient preserved the “what” does not rest only on his good definition of a multiplication but on his telling explicitly (upon explicit instructions), in a step-by-step fashion, what had to be done (i.e., he knew what carrying is and in fact could do it in addition!). It is certainly possible to work out the right procedure on a reasoning basis only, but it would be painstaking and probably not within everybody’s ability. It is contended here that normal calculators (as well as PN) do not do it this way: what they do is to apply the operation of carrying where an overlearned procedure dictates. In any case carrying were not the only errors.

Besides the above listed errors, PN also made errors that have been already described in right hemisphere spatial acalculia: (a) omission of a digit on the left side; (b) omission of the auxiliary line; (c) column confusion. These errors can be easily isolated and distinguished from the ones listed above, and can certainly be considered as due to failure in the spatial layout specific for multiplication, once generic across-the-board spatial problems are ruled out.

Errors due to across-the-board spatial difficulties cannot indeed be claimed to be at the basis of all PN’s problems. While, in fact, it is not possible to exclude the influence on PN’s performance of spatial disorders of the type known to affect right hemisphere patients, it is possible to contend that such generic disorders of to space or of space representation are not sufficient to produce all the observed types of errors.

In fact, a generalised disorder in the attention to space would hardly produce systematic errors, but would instead leave the patient helpless in his space exploration: as a consequence the display of the operation would reflect more or less chaotic attempts equally affecting every portion of the operation, and not just specific steps (why, indeed, should the spatial arrangement of some steps be systematically more resistant to attention problems?). PN was not particularly prone to attention problems.
Equally, PN’s deficit with multiplication procedures could not possibly stem from the more restricted attention disorder described under the heading of “monitoring” deficit (Semenza et al., 1997); in fact PN was consistent in his errors, always ended the operation, and his mistakes happened only at certain levels and did not increase with the progress of the operation.

Unilateral spatial neglect (a likely symptom in a right hemisphere lesion, but clinically absent in every form when the second series of tasks was administered) would also have affected complex, multi-digit addition and subtraction (carrying is certainly not more complex than borrowing), but these operations were not disturbed at least at the time of the second evaluation. A possible objection may be that multiplication is more demanding and likely to uncover neglect in a milder form. However, in multiplication PN makes the same kind of mistake even in simpler operations like (234 × 6). Why, in this case, carrying in addition differs from carrying in multiplication? Only because carrying in multiplication happens in a different context: according to the learned spatial procedure one has to operate in a different space display: there is no mathematical or conceptual difference whatsoever and, as just said, there are cases where multiplication cannot be considered more spatially demanding; PN’s main problem is rather that he forgot where to apply the operation step.

There is, however, further proof against a significant contribution of neglect to the whole of PN’s spatial symptomatology. In fact, a distinctive type of error committed by PN consisted of explicitly writing the carry to the leftmost side of the multiplicand as well as of the partial products. The presence of a left sided neglect would dictate writing carry notes to the right, or, however, not on the left. This behaviour cannot derive by an automatic procedure because it is not something that is learned. To be true (see above) PN also made a few omissions of digits in the left side, but their number was negligible and this type of error was virtually absent at the time of the second administration.

Another hypothesis to keep into account is that PN’s errors may have originated from a disturbance of a spatial working memory component, a sort of “calculation sketching pad”. While it may be impossible to entirely exclude the influence of a limitation in such putative mechanism, that may certainly contribute to PN’s difficulties, this is still an unlikely explanation. In fact PN was, though at the lower level, within norms at Corsi’s test, testifying a sufficiently efficient spatial memory span; furthermore his errors were not casually dispersed.

In summary, the best explanation for PN’s problems cannot be found in a generic deficit of spatial processing or of spatial attention. Such deficits may not be entirely ruled out for PN. Indeed, there would never be enough testing for a sceptical neuropsychologist to exclude these generic sources deficits in any given patient. However, negative symptoms should not be the only ones guiding the interpretation of neuropsychological patients’ performance. More information is carried by positive symptoms. PN’s difficulty with multiplications may indeed reflect his lack of generic resources but his non-casual errors point the fact that he cannot (unlike normals that have been thought to do so) make up for his deficit with the spatial framework or spatial layout that helps in conducting complex operations correctly to an end. In other words, if PN had an intact knowledge of the spatial layout, he would be able to compensate for the calculation problems he displays, irrespective of their origin. The observation of such a case makes it compulsory analysing future cases at this level of detail. It will be important trying to answer questions as the following: to what extent to know “where” certain operation steps should be displayed in writing is based on spatial cognition? How can one distinguish between knowledge of these “spatial facts” or “spatial procedures” and visuo-spatial representations? How optional is to resort to spatial knowledge in carrying out a complex arithmetical operation? Most of the analyses made in this study have never been applied before and thus the evidence collected in this particular case may not be entirely compelling. It is nevertheless intriguing to suggest that patients like PN might have difficulties in relying on a stored layout representation specific to multiplication. This representation would be normally activated in a (quasi-)automatic fashion in order to support spatially arranged complex calculation and dispense with more demanding controlled processing based on logical criteria. As already mentioned, a representation of this sort has been postulated by Dehaene and Cohen (1995) along with a preliminary description. PN’s difficulties in the carry procedure match well with this assumption, because PN knows that a retrieved fact has to be decomposed but fails to add it to the next fact in the right place. Thus, PN seems to fail in intermediate and “border steps” in a multiplication, while there is evidence that he knows what steps to make. All these errors may be typified as resulting from the attempt to compensate to the lack of a guiding spatial scheme.

A speculative but likely conclusion is therefore that, vis-à-vis his pattern of performance in multiplication procedures, PN does know what, when and how to carry out the various steps, but he does not know where. What he may lack is a spatial schema of multiplication. This putative spatial schema can be seen as the sum of spatial markers, which might be necessary to most calculating subjects, especially in intermediate steps and or “border steps”. Such schema might thus help in keeping the single sub-steps together by representing the information of where exactly each sub-step should be placed. Next case studies on patients similar to PN should be aimed at testing these hypotheses. Equally, PN’s case points out to the necessity of collecting further evidence in the learning domain: learners’ strategies to cope with logical and spatial aspects of complex multiplication must be further disentangled. As some authors active in the first half of last century (Ehrenwald, 1931; Peritz, 1918) had suspected, on the basis of observations in the clinical and developmental domain as well as on phenomenological considerations (inspired by works such as those of Bergson (1911) or Wertheimer (1912)), there might, after all, be a primary spatial acalculia. Whether affecting functions that are normally stored in the right hemisphere, as PN’s case seems to suggest, is still an open question.

Acknowledgements

We wish to thank Prof. Paolo Di Benedetto and Dr. Emanuele Biasutti of the Istituto di Medicina Fisica e Riabilitazione “Gervasutta”, Azienda per i Servizi Sanitari, no. 4 “Medio Friuli”,
Italy for their support throughout this study. The present research has been supported by the European Union (Marie Curie Action Contract “NUMBRA” 504927) and by a MIUR grant 2005 to Carlo Semenza.

References


