INTRODUCTION

When considering various cultures, human body parts have been used as aids to counting in the development of some number systems. Also, the significance of fingers has been highlighted particularly in the context of early numerical development from a neuropsychological perspective by Butterworth (1999). A variety
of functions have been associated with the fusiform gyrus which is a structure located in the superior temporal region of the brain. Recognition of Arabic numerals presented visually involves the fusiform gyrus, particularly the left as highlighted by Ramachandran and Hubbard (2005) with reference to Pesenti et al. (2000) and Rickard et al. (2000). In relation to neuroimaging research findings McCandliss et al. (2003) have suggested that the process of abstract visual word form perception is associated with a cortical area located within the left fusiform gyrus. Also, they note with reference to Haxby et al. (2000) that evidence has suggested there is activation in this area resulting from object and face recognition. Further discussions concerning the notion of a visual word form area and related issues have been given by Cohen and Dehaene (2004) and Price and Devlin (2003, 2004). For developmental perspectives in relation to facial perception and the fusiform gyrus reference could be made to Passarotti et al. (2003), Gathers et al. (2004), Aylward et al. (2005) and Scherf et al. (2007).

Facial features are some aspects of the body that have been noted in relation to number systems and they have been considered particularly by certain investigators in connection with numerical disabilities. Badian (1983) considered that a deficit in visual attention to exact detail as suggested in young children’s human figure drawings might provide information concerning later numerical abilities. In Badian’s discussion concerning those who might be described as dyscalculic, the omission of the nose was noted and sometimes the inclusion of an incorrect number of fingers.

Also, findings from research by Lawson (2000c, 2001a) with first-year junior-aged children suggested that directing their attention to specific arithmetical details and associated numerical concepts or, alternatively, particular omissions and poorly depicted aspects concerning their drawings of people enhanced their development and performances in both areas. Additionally, in another study by Lawson (2001b), first-year junior-aged pupils practised a task involving subitizing which concerns the fast apprehension of small numerosities. The numerosities were associated with dots for some presentations and for others with schematic facial features. The results suggested a facilitative effect in relation to the inclusion of details in their human figure drawings as well as more realistic depictions.

The findings presented by Noël (2005) indicated that in first-grade children finger gnosis was a good predictor of numerical skills. Also, this occurred for left–right orientation. Finger gnosis predicted performance equally on numerical tasks whether or not they depended particularly on finger representation or on magnitude representation. Also, Rusconi et al. (2005) found that in adults repetitive transcranial magnetic stimulation (rTMS) over the left angular gyrus disrupted finger gnosis and number magnitude processing. In a more recent study by Gracia-Bafalluy and Noël (2008) with first grade children, it was suggested that finger differentiation training improved performances in finger gnosis, subitizing, counting representations of raised fingers and ordinality involving Arabic digit processing. In the finger gnosis assessment the children were required to differentiate their fingers when touched and without visual cues. The specific
task had been used by Noël (2005). Also, as part of the finger gnosis assessment the children were involved in a ‘Draw a Man’ and a ‘Draw a Hand’ task. On the latter, participants in the finger differentiation training group demonstrated enhanced performances in comparison with those in the control group who had received story comprehension training, after individuals in each group had received their specific interventions. The possibility was considered of improvements in internal representations concerning the fingers and hands but with no change for those involving the whole body.’

Additionally, Sato et al. (2007) used TMS to investigate changes in excitability relating to hand muscles of right-handed adult participants during their involvement with a visual, odd or even parity judgment task. It concerned the Arabic numerals from 1 to 9, with the exclusion of 5 and there was not a requirement for counting. No modulation was indicated for the left-hand muscles, but an increase in amplitude concerning the motor-evoked potentials was demonstrated for the right-hand muscles. The increase was apparent only for the smaller numbers from 1 to 4 as opposed to the larger numbers from 6 to 9. Hence a neural association was suggested between hand/finger and numerical representations. Also, in a study by Andres et al. (2007) right-handed adult female participants were involved with a counting task requiring the use of numbers or letters of the alphabet to enumerate items. Changes in corticospinal activity (CS) were assessed via TMS. The results indicated an increase in CS excitability relating to hand muscles when the participants were performing the task using numbers or letters. The increase in CS excitability was identical when the activity involved small arrays of dots from 1 to 4 as opposed to large arrays comprising 9 to 12 dots. No changes were apparent in relation to the arm and foot muscles. Hence, the involvement of hand motor circuits was considered for tasks requiring items to be placed correspondingly with the elements of any ordered series.

In research by Riggs et al. (2006), subitizing was investigated in relation to tactile perception via the simultaneous stimulation of the fingertips on both hands of adult participants. A discontinuity in accuracy was shown with near perfection for one to three fingers and severe impairment for four to six fingers. Also, there was a discontinuity in the naming-times slope for one to six fingers. Hence it was concluded that subitizing was apparent in tactile perception within this context. In a different context but still involving touch, the results of research by Gallace et al. (2006) suggested that adults were able to discriminate to a certain extent between different numbers of tactile stimuli when multiple tactors were activated simultaneously across the surface of the body. However, the accuracy of tactile numerosity judgments decreased considerably with relatively modest increases in the number of activated tactors. The participants indicated the ability to estimate differences in the number of tactors comprising the pattern presented on the body, but this was less so for the correct perception of information concerning the actual quantity. There was no indication of a discontinuity in the slope corresponding to the reaction time or the error data, so subitizing was not suggested by the results. Also, reference could be made to Gallace et al. (2007a) in relation to numerosity judgments concerning visual and tactile stimuli and for
detailed discussions of cutaneous tactile stimulation within a multisensory context to Gallace and Spence (2008) and Gallace et al. (2007b).

In a functional magnetic resonance imaging (fMRI) study by Thompson et al. (2004), adult participants viewed videos of a hand with small numbers being represented by finger movements or faces where lip movements presented the same numerical information. Control stimuli comprised the same hand half opening and closing or the same mouth opening and closing for each comparable test item. Activation relating to lip reading of numbers was apparent in the left posterior superior temporal sulcus (STS). Identification of numbers conveyed by the fingers was indicated by preferential activation in the left inferior parietal region (IPR). Also, activation in relation to numbers via the presentation of finger stimuli was shown in the right IPR. This response was not specific to the number representation via finger stimuli, and there was a small area of common activation in the right IPR associated with the identification of numbers in the finger and lip numerical presentations. In relation to their findings the authors referred to an fMRI study by Eger et al. (2003). Adult participants were asked to respond to target items within each category of numerals, letters and colors presented in the visual and the auditory modalities. Specifically, this nonarithmetical processing of numerals in comparison with letters and colors activated a bilateral region in the horizontal intraparietal sulcus for the visual and auditory modal presentations. Thompson et al. (2004) noted that this supramodal number region as termed by Eger and colleagues was proximal to the intraparietal regions activated by fingers representing numerical information in their study.

As indicated below concerning the presentation here, the fingers were used particularly in relation to the tactile aspects of the remedial technique as the participants felt the shape of each solid Arabic numeral. Also, they might use fingers overtly or covertly for counting when necessary although the extent to which they were used varied as different strategies and direct recall became available increasingly during the development of their numerical abilities. Apart from moving their lips as they said the number words, they watched my lips move sometimes during the interactions and including the times when I spoke the number words. Also, there were no restrictions on the movement of the hands except for the specific application as described for the multisensory technique for learning the multiplication tables. The importance of the hands not only in relation to their use for the representation of the attentional and declarative aspects of cognitive activities but in terms of providing support within a dynamic context for the procedural aspects of these activities has been discussed in some detail by Carlson et al. (2007). Specifically they considered the use of the hands in elementary arithmetic tasks and emphasized the importance of studying the embodiment of cognition. For a specific discussion concerning the embodied mind and mathematics, reference could be made to Lakoff and Núñez (2000).

The angular gyrus which has been highlighted above is situated within the inferior parietal lobule and is located at the junctions of the temporal, parietal...
and occipital lobes. Generally this region has been considered to be involved in the multisensory convergence of information from touch, hearing and vision to facilitate the construction of higher-level abstractions (Ramachandran and Hubbard, 2005). It would appear that at birth we are predisposed to manage multisensory input in an environment which provides information mainly of a multimodal nature. Also, multisensory processing and integration have been of particular interest in association with theories and interventions concerning some specific conditions such as those within the autism spectrum (Iarocci and McDonald, 2006). In relation to numerical development some studies, for example, Kobayashi et al. (2005) and Jordan and Brannon (2006) have suggested that infants are able to relate sets with small numerosities when they are presented in different sensory modalities. Consequently, it would seem reasonable to consider teaching/learning approaches that emphasize multisensory stimulation with the aim that compensatory mechanisms might be facilitated for those individuals who might experience difficulties in certain areas.

The first two children discussed here had been selected originally because they had experienced difficulties in learning the multiplication tables. Before their involvement in a remedial program to help them overcome these specific problems and to promote more mature numerical development, they were assessed in terms of their levels of cognitive and psychosocial functioning as well as their attainments in literacy and numeracy. The third child given in this presentation had problems also in learning the multiplication tables so it was appropriate to involve him in a similar remedial program. However, because of social and communication difficulties he participated also in other assessments and interventions some of which had been designed specifically by the present author.

As part of the assessment, items were developed in relation to the Dyscalculia Test Battery described by Macaruso et al. (1992) and there were some specific additional tasks too. Their battery was based on the cognitive model of number processing and calculation suggested by McCloskey et al. (1985) and supported by evidence of specific deficiencies in adults with acquired dyscalculia. In this model a number processing section comprised distinct components for comprehension and production, lexical and syntactical elements and separate Arabic and verbal number processes. Another aspect involved knowledge relating to number facts including the meaning of signs and knowledge of tables. Also, an area concerned procedural knowledge including the necessary algorithms for carrying out exercises involving the four arithmetical operations.

Difficulties might be experienced by individuals described as dyscalculic in any of the spheres concerning numerical processing, knowledge of number facts and procedural knowledge. Temple (1989, 1991, 1994a, b, 1997a, b) discussed the main characteristics of some young people who might be described as dyscalculic within a developmental cognitive neuropsychological context and with some considerations support was given for the model proposed by McCloskey and his colleagues. Reference could be made to Campbell and Epp (2005) who have reviewed studies and considered challenges in relation to aspects of the
model and possible modifications. Additionally, Kaufmann and Nuerk (2005) have highlighted issues concerning the use of adult numerical cognitive models in relation to children and adolescents, and their discussion has included aspects relating to the model by McCloskey and his colleagues.

Also, a recent discussion concerning dyscalculia including developmental and acquired considerations associated with various hypotheses as well as implications for educational aspects and interventions has been presented by Wilson and Dehaene (2007). They highlight the significance of the angular gyrus and a suggested impairment involving this region and associated with difficulties in the learning and retrieval of arithmetical facts, especially relating to multiplication. Additionally, reference could be made to the various discussions concerning mathematical learning difficulties and disabilities in the book edited by Berch and Mazzocco (2007). Specifically, Simon and Rivera (2007) emphasize caution concerning the findings of neuroanatomical correlates relating to numerical cognition in neuroimaging studies involving adults and interpretations concerning children and adolescents as differences in neural activity might be found in young people with typical and atypical presentations at various stages in their development. Also, there are a limited number of published neuroimaging investigations concerning numerical development involving children and adolescents.

In an fMRI study by Kawashima et al. (2004), young people of both sexes, aged 9–14 years and adults, men and women aged 40–49 years were involved in mental calculation tasks concerning addition, subtraction and multiplication with single-digit operands. Brain activation was demonstrated in the prefrontal, intraparietal, occipital and occipito-temporal cortices for both groups of participants and in relation to when they performed each of the arithmetic operations. The young people who took part in my research were involved with learning items from the multiplication tables, and some of these had single-digit operands whereas others involved two-digits. Also, although the number of items that they could retrieve directly increased with training, if they were unable to recall an answer immediately they might recall another item and use mental addition or subtraction to obtain the correct answer. Additionally, because of their difficulties and the nature of the training they experienced the multiplication items in a variety of modalities. In the study highlighted above the items were presented on a computer screen, and the participants were instructed to perform the tasks mentally and not to vocalize or move parts of their bodies.

Generally, I have noted the results of some studies with children, adolescents and adults which seemed of interest and relevance to the discussion presented here. Overall, assumptions have not been made in relation to these findings and the location of neural activation in the young people who have participated in my projects. Also, I implemented items which I had developed in relation to a particular dyscalculia test battery as the resulting assessment tool was expected to be helpful in highlighting specific numerical difficulties experienced by the young participants in my studies.
In relation to some of the terms and ideas used by myself and in various studies, reference could be made to Shalev (2007) for a discussion of aspects concerning developmental dyscalculia in the context of definitions, assessments and prevalence. Shalev and Gross-Tsur (1993) considered a marked impairment of arithmetical skills combined with a lack of response to educational interventions as the criteria for developmental dyscalculia in the children involved with their investigation. Specific arithmetical impairments had been noticed by staff or parents concerned with the young people described here, and they had not responded to remedial interventions or only to a limited extent. The case reports presented here give analyses of the difficulties experienced by three boys. The findings are related to other investigations concerning children’s numerical development, case studies of young people with developmental dyscalculia and research involving individuals or groups of adults with acquired dyscalculia.

Mainly, information obtained from the Dyscalculia Test Battery Assessment is presented here but a few points will be noted in relation to some specific numerical tasks which were administered to the children as part of the whole assessment. The first two cases, namely Child A. and Child M., were involved with their assessments and remedial programs some time ago, whereas Child L. was seen more recently and he participated in other sessions concerned with different aspects of his development because of his disposition as indicated below. Consequently, some different tests or more revised versions of some assessments were used for Child L. as opposed to the other two boys. Detailed presentations of the programs for Child A. have been given by Lawson (1995b, 2000b) and for Child M. by Lawson (2000b) with summaries by Lawson (2001a). Also, certain aspects relating to these two children reported here were presented initially by Lawson (2002). A comprehensive presentation concerning the assessments and interventions for Child L. has been given by Lawson (2005).

Initially, a detailed description of the Dyscalculia Test Battery is given as well as the remedial technique used to help children who experienced difficulty with certain types of addition problems. Also, a Multisensory Remedial Approach for Learning the Multiplication Tables developed by the present author is described here. This technique involved the visual, auditory, tactile and kinesthetic modalities and is similar in certain respects to that described for reading and spelling by Bryant and Bradley (1985). Also, the modified procedure for learning the operation names, words and symbols is given in this presentation.

As discussed by Fogassi and Gallese (2004), multisensory integration is a pervasive characteristic of cortical regions concerned with motor planning and control. Cortical premotor regions possess sensory features and posterior parietal regions that have been considered association areas possess motor features. Parietal regions which are connected with frontal regions jointly comprise cortical networks for the processing and integration of multisensory data for the execution of action and the representation of the environment in which the action occurs. The multisensory technique that I developed involved a motor component and this aspect is a feature of other approaches described below.
Specifically, my approach shares some similarities as well as differences with TouchMath (Bullock, 2002) which has been highlighted by Naglieri and Pickering (2003). This is a multisensory method for learning computation with applications in relation to all four arithmetical processes. Also, my approach has certain aspects in common with the ARROW technique (Lane & Chinn, 1986; Lane, 1992) and cited by Chinn and Ashcroft (1998). ARROW, which is an acronym for Aural-Read-Respond-Oral-Written is a multisensory teaching/learning approach. The learner’s own voice is replayed on tape and connected with skills in writing, listening and speech in a set of processes concerning spelling, comprehension and reading books. Lane and Chinn (1986) and Chinn and Ashcroft (1998) describe the technique in relation specifically to learning the multiplication tables.

The author would like to emphasize that the relatively simple techniques concerning addition and multiplication discussed here were accompanied always by instructional support aimed at integrating and promoting the development of numerical concepts and knowledge of arithmetical procedures with the acquisition of number facts. The iterative nature of factual, procedural and conceptual knowledge concerning numbers has been emphasized in a developmental context by Baroody (2003) and highlighted in relation to the rehabilitation of acquired calculation and numerical processing disorders by Lochy et al. (2005). Also, Zamarian et al. (2007) have emphasized the influence of interrelationships concerning the different aspects of numerical knowledge in promoting the development of meaningful and efficient processing in this area. In relation to my participants, often the children offered comments and ideas spontaneously which indicated the nature of their understanding and these could be developed further. For those who were less forthcoming, they could be prompted to question and think about various aspects concerning the items that had been problematic.

Also, although I refer to Arabic numerals I do acknowledge consideration in relation to the Indian numerical symbolic notation as highlighted, for example, by Ifrah (1998). Additionally, McCloskey et al. (1985) refer to Arabic and verbal numbers. The distinction between numbers and their symbolic representation via the use of the term ‘numerals’ has been highlighted by McCloskey (1992) in relation specifically to Arabic numerals for numbers represented symbolically in digit form as well as spoken and written verbal numerals for numbers represented as words. Hence, I have used the term ‘numerals’ in connection with the dyscalculia model which I have described here. However, in line with everyday usage of the term ‘number’ I might have used this terminology when making a request to a child, for example, to find a number when referring to solid plastic numerals in Arabic form for the Tactile Recognition and Naming of Numerals task. Also when I refer to number(s) in the discussion, the exact representation and any conceptual associations should be clear from the context.
A DYSCALCULIA TEST BATTERY

NUMERAL PROCESSING TASKS

Magnitude Comparison

Arabic Magnitude Comparison. Two Arabic numerals were presented to the child who on request pointed to the larger or smaller number. The test items included units, tens, hundreds and thousands. There were 10 comparisons for the units with the numbers ranging from 0 to 9 and 10 comparisons each for the tens in the range 10–19 and in the range 20–99. For the hundreds and the thousands there were five comparisons each that were selected respectively within the ranges 9,999. The items were chosen so that a broad selection was offered and included odd and even numbers and those involving zeros. Also, they were written and presented on white cards.

Spoken Verbal Magnitude Comparison. Two verbal numerals were presented in spoken form and the child indicated which was larger. On a piece of white card, two squares had been drawn one above the other. The assessor pointed to the top square while reading the first number and to the bottom square when reading the second number. When asked to respond to the larger or smaller number the child responded by pointing to the appropriate square. The test items were the same as those used for the Arabic Magnitude Comparison tasks.

Written Verbal Magnitude Comparison. Two written verbal numerals were presented to the child and he/she pointed to the larger or smaller number as requested. The test items were the same as those used for the Arabic Magnitude Comparison Tasks except that they were in written verbal form.

Transcoding Tasks

On these tasks the child was asked to perform six possible conversions among Arabic, spoken verbal and written verbal numerals. All of the test items used were the same as for the magnitude comparison tasks.

Transcoding Arabic Numerals to Spoken Verbal Numerals. Arabic numerals were presented visually and the child read each numeral aloud.

Transcoding Spoken Verbal Numerals to Written Verbal Numerals. Verbal numerals were dictated and the child wrote each numeral in verbal form.

Transcoding Spoken Verbal Numerals to Arabic Numerals. Verbal numerals were dictated and the child wrote each numeral in Arabic form.

Transcoding Arabic Numerals to Written Verbal Numerals. Arabic numerals were presented visually and the child wrote each numeral in verbal form.

Transcoding Written Verbal Numerals to Spoken Verbal Numerals. Written verbal numerals were presented and the child read each numeral aloud.

Transcoding Written Verbal Numerals to Arabic Numerals. Written verbal numerals were presented and the child wrote each numeral in Arabic form.
Additional Item: Tactile Recognition and Naming of Numerals. Solid plastic Arabic numerals from 0 to 9 were placed under a cloth and the child was asked to find individual numbers at random and to name them one at a time while they were under the cover. Also, the child was asked to retrieve named numbers that were chosen individually at random from under the cloth.

**CALCULATION TASKS**

**Operation Symbol and Word Comprehension**

*Operation Symbol Comprehension.* This task comprised items probing comprehension of the operation symbols for addition, subtraction, multiplication and division. For each item an operation name was presented visually and aurally. Arithmetic problems with identical operands but different operation symbols were presented visually. The child’s task was to point to the problem corresponding to the specified operation. The test items included units, tens and hundreds. There were four items involving two numbers each in the units for assessing comprehension of the four arithmetical operations. Also, there were four items involving two numbers each, with a number in the 10–40 range and a number in the 1–9 range and another group of four items each involving two numbers where one number was in the range 100–400 and the other number was in the range 10–60.

*Operation Word Comprehension.* In this task, items were used which probed comprehension of the spoken operation words ‘plus’, ‘minus’, ‘times’ and ‘divided by’. An operation name was presented visually and aurally for each item. Then an arithmetic problem was dictated and the child indicated whether or not the problem corresponded to the operation name. A yes/no procedure was used for the response. The test items were selected from the Operation Symbol Comprehension tasks.

**Written Arithmetic Tasks**

These tasks were used to investigate the retrieval of arithmetic facts and execution of the calculation procedures for addition, subtraction, multiplication and division. For Child A. and Child M. three arithmetic tests were administered, specifically the first 25 questions of the Graded Arithmetic–Mathematics Test (Junior) by Vernon and Miller (1976), the Basic Number Diagnostic Test by Gillham (1980) and the British Ability Scales (BAS)-Basic Number Skills by Elliot et al. (1983). Child L. was administered both sections of the Weschler Objective Numerical Dimension (WOND)-Mathematical Reasoning (MR) and Numerical Operations (NO) (Rust, 1996). For the MR section, the questions are seen visually by the child and they are read to the participant. Some items require a written answer but for many questions an oral response is required by the individual and a few items require the child to point in response to a question. On the NO section the answers to all of the questions have to be written.

*Additional Item: Specific Rectangular and Non-Rectangular Addition Exercises.* The children were administered certain types of rectangular and
non-rectangular addition questions, groups (A, B) and (A1, B1) which I had prepared myself and the necessary teaching involving groups (A2, B2) and re-assessment with groups (C1, D1) as required if they experienced difficulty with these exercises. The non-rectangular addition problems had been highlighted by Friend (1979) and confirmed by Lawson (1986, 1989) as being a source of particular difficulty for some children, and these studies had noted three specific types of errors. Examples of items in the different groups and particular errors are shown below. The terms rectangular and non-rectangular referred to the digital arrangement in the addends. The non-rectangular questions contained a single-digit in the far left column and there was no carry to be added into this column. I presented the children with rectangular exercises that matched the non-rectangular questions in terms of the number of columns and rows and no carrying was required in the calculations. The difficulties concerning the non-rectangular problems had been considered in relation to conceptual issues concerning the join of a single set and the sum of a single number. The questions could be completed accurately after instruction with a simple technique involving a zero being placed in the space in the far left column and a discussion to clarify the associated conceptual basis for the procedures (Lawson, 1990, 1995a, 2000a, 2001a).

Examples of the rectangular and non-rectangular addition exercises given in each of the groups:

$$\begin{array}{c}
\text{A} & \begin{array}{c}
4 \\
+ 2 \\
\hline
\end{array} & \text{B} & \begin{array}{c}
3 \\
\hline
\end{array} \\
\text{A1} & \begin{array}{c}
6 \\
+ 3 \\
\hline
\end{array} & \text{B1} & \begin{array}{c}
5 \\
\hline
\end{array} \\
\text{A2} & \begin{array}{c}
6 \\
+ 0 \\
\hline
\end{array} & \text{B2} & \begin{array}{c}
5 \\
\hline
\end{array} \\
\text{C1} & \begin{array}{c}
3 \\
\hline
\end{array} & \text{D1} & \begin{array}{c}
4 \\
\hline
\end{array} \\
\end{array}$$

Examples of the specific types of errors on the non-rectangular addition exercises:

**Type 1:** A number is added from another column e.g.

$$\begin{array}{c}
\text{6} & \text{3} \\
\hline
\text{8} \\
\text{5}
\end{array}$$

**Type 2:** The single digit is added into the column on the right e.g.

$$\begin{array}{c}
\text{6} & \text{3} \\
\hline
\text{1} \\
\text{1}
\end{array}$$
Type 3: The odd digit is ignored e.g.

\[
\begin{array}{c}
63 \\
+ 2 \\
\hline
5
\end{array}
\quad + \quad
\begin{array}{c}
561 \\
+ 31 \\
\hline
92
\end{array}
\]

Oral Arithmetic Tasks

These tasks probed arithmetic fact retrieval and involved the four calculation procedures. Each problem was presented aurally and the child said the answer aloud. Five questions were presented with each item involving two numbers in the range 0–9 for the arithmetic operations addition, subtraction, multiplication and division. Also, Child A. and Child M. were administered the Arithmetic subtest of the Weschler Intelligence Scale for Children–Revised UK (WISC-R UK), Weschler (1976). Child L. was administered the Arithmetic subtest of the Weschler Intelligence Scale for Children–Third Edition UK (WISC-III UK). For both versions of the Arithmetic subtest, the children were administered aurally presented word problems requiring oral responses.

Additional Item: Recitation of Multiplication Tables. The child was asked to recite any of the multiplication tables or any items that he/she was able to recall.

It should be noted that division items were included in the calculation section here whereas they were not specified in the dyscalculia test battery described by Macaruso et al. (1992).

A MULTISENSORY REMEDIAL APPROACH FOR LEARNING THE MULTIPLICATION TABLES

As indicated earlier the teaching approach used to help the children learn the multiplication tables involved a multisensory method with an emphasis on four different sensory modalities. Prior to the remedial sessions, cards had been prepared on which there were written numerals in verbal and Arabic form. Also, there were cards for the four arithmetic operations and expression for equality in verbal and symbolic form. Plastic characters were available that represented Arabic numerals and the four arithmetic symbols and there was one for equality. For some multiplication items the children displayed the word and symbol cards and solid characters in an appropriate order. As they placed the items, they said the names of the numerals and arithmetic operations out aloud and similarly they acknowledged equality. The cards could be used as aids for those with limited literary skills or until the child became more proficient at accessing items, but they were not implemented for every exercise. For all multiplication exercises the children wrote the number words and the arithmetic operation words and they named them aloud as they wrote the items. Then they wrote out the numerals in Arabic form and the arithmetic operation symbols and again they said the names out aloud as they wrote the items. Also, the children placed solid
plastic characters under the written Arabic numerals and operation symbols or on a board if the characters were magnetic for each multiplication item and they said the names out aloud. Then they felt and named these solid characters with their eyes closed and with them open. When they repeated the procedure for other table entries they removed only the cards if used and the solid characters that were not needed to form the new items. If a child showed confusion particularly with arithmetic operation names, words and symbols a similar procedure was carried out using the same operands in each case but with different operations.

Example of the items and procedure used for learning the multiplication tables:

<table>
<thead>
<tr>
<th>three</th>
<th>times</th>
<th>four</th>
<th>equals</th>
<th>twelve (written)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>×</td>
<td>4</td>
<td></td>
<td>12 (written)</td>
</tr>
<tr>
<td>3</td>
<td>×</td>
<td>4</td>
<td></td>
<td>12 (solid characters)</td>
</tr>
</tbody>
</table>

Example of the items and procedure used for learning the operation names, words and symbols:

**Multiplication:**

<table>
<thead>
<tr>
<th>ten</th>
<th>times</th>
<th>five</th>
<th>equals</th>
<th>fifty (written)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>×</td>
<td>5</td>
<td></td>
<td>50 (written)</td>
</tr>
<tr>
<td>10</td>
<td>×</td>
<td>5</td>
<td></td>
<td>50 (solid characters)</td>
</tr>
</tbody>
</table>

**Addition:**

<table>
<thead>
<tr>
<th>ten</th>
<th>plus</th>
<th>five</th>
<th>equals</th>
<th>fifteen (written)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>+</td>
<td>5</td>
<td></td>
<td>15 (written)</td>
</tr>
<tr>
<td>10</td>
<td>+</td>
<td>5</td>
<td></td>
<td>15 (solid characters)</td>
</tr>
</tbody>
</table>

**Subtraction:**

<table>
<thead>
<tr>
<th>ten</th>
<th>minus</th>
<th>five</th>
<th>equals</th>
<th>five (written)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>−</td>
<td>5</td>
<td></td>
<td>5 (written)</td>
</tr>
<tr>
<td>10</td>
<td>−</td>
<td>5</td>
<td></td>
<td>5 (solid characters)</td>
</tr>
</tbody>
</table>

**Division:**

<table>
<thead>
<tr>
<th>ten</th>
<th>divided by</th>
<th>five</th>
<th>equals</th>
<th>two (written)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>÷</td>
<td>5</td>
<td></td>
<td>2 (written)</td>
</tr>
<tr>
<td>10</td>
<td>÷</td>
<td>5</td>
<td></td>
<td>2 (solid characters)</td>
</tr>
</tbody>
</table>

**CHILD A.**

Initially when A. participated in the project he was aged 11 years 1 month and he was in the last year of his primary education. He was expecting to move on to secondary school for the next academic year and some of the comments made here relate to discussions that I had with A. during his first year at secondary level. He had attended mainstream schools throughout his school life and he had not received any form of specialist educational support. As indicated earlier, A. was involved with a variety of assessments but mainly information is presented concerning the Dyscalculia Test Battery. However a few points involving specific numerical tasks are mentioned in relation to this. On the
WISC-R (UK), Weschler (1976) his performance was not strong on the Arithmetic subtest which involves aurally presented word problems with various arithmetical operations that have to be solved mentally by the child and require an oral response. Also, he showed particular difficulty on Digit Span and this was reflected by his limited performance both on the Forward and on the Backward Digit Span tasks. Hence, it was suggested that A. might have some difficulties concerning the storage of numerical data and the manipulation of stored numerical information as discussed by Hoard et al. (1999) in relation particularly to performances on Backward Digit Span within the context of working memory.

**DYSCALCULIA TEST BATTERY ASSESSMENT**

**NUMERAL PROCESSING TASKS**

**Magnitude Comparison**

*Arabic Magnitude Comparison.* All items were completed correctly.

*Spoken Verbal Magnitude Comparison.* All items were completed correctly.

*Written Verbal Magnitude Comparison.* All items were completed correctly.

**Transcoding Tasks**

*Transcoding Arabic Numerals to Spoken Verbal Numerals.* A. was required to read aloud each Arabic numeral presented in a visual format. This was a reading task of a similar type to that used by Power and Dal Martello (1997) with Italian children aged 7 years. Various errors were made by these children who were much younger than A. whereas he was correct on all items.

*Transcoding Spoken Verbal Numerals to Written Verbal Numerals.* All items were completed correctly.

*Transcoding Spoken Verbal Numerals to Arabic Numerals.* A. made one error when he wrote 9,804 in response to the spoken numeral ‘nine thousand and thirty-four’ (9,034). The dictation task used here was of a similar form to that used by Power and Dal Martello (1990) with Italian children aged 6–8 years. Generally in the area of numeral processing, the lexical processing component is concerned with the processing of individual number words and digits whereas syntactic processing concerns the relationships among the elements that comprise a numeral. Hence, as highlighted by Power and Dal Martello (1990) in relation to a dictation task, a response for example to the spoken verbal numeral ‘three hundred and sixty-five’ might involve a lexical error as apparent in 364 as opposed to the correct answer 365. A syntactical error might involve an incorrect number of zeros as in responses such as 3,065 or 30,065 as opposed to the correct answer 365.

Following Power et al. (1978), the investigators gave a theoretical formulation of these errors in which it was proposed that in the production of an Arabic numeral like 365, the numerals 300 and 65 need to be combined via a string operation which they termed ‘over-writing’. Children who had not yet acquired
competence with this operation tended to rely on concatenation. Generally, they found that syntactical errors were much more frequent than lexical errors and when considering the types of syntactical errors the insertion of zeros was predominant as opposed to the incorrect ordering of digits, such as 563 or 653 instead of the correct response of 365, in the example given above. Also, the insertion of zeros was not apparent for numbers less than hundred. The error made by A. in response to a spoken verbal numeral, when he wrote 9,804 instead of 9,034, appeared to have both syntactical and lexical components as there seemed to be a reversal of two digits and then a digit was written incorrectly. Temple (1989, 1997a, b) described an 11-year-old child who produced incorrect digits when he was required to read Arabic numerals or write Arabic numerals that had been dictated but he demonstrated accurate syntactic processing. More errors were made in relation to reading number words in comparison with Arabic numerals but they were of a similar type. As indicated above and below respectively, A. did not make any errors when reading Arabic numerals or number words.

Transcoding Arabic Numerals to Written Verbal Numerals. A. made two errors when transcoding visually presented Arabic numerals to written verbal numerals with the written verbal response of ‘twelve thousand and seventy-eight’ being given instead of ‘one thousand two hundred and seventy-eight’ or ‘twelve hundred and seventy-eight’ and ‘five thousand and sixty-three’ being written instead of ‘five thousand six hundred and three’. In the first incorrect response written by A., he appeared to have transcoded erroneously the first part of the numeral from ‘twelve hundred’ to ‘twelve thousand’ so he made a ‘Stack’ error by retaining the 12th position within the stack but the information relative to the stack itself was altered, that is from the hundreds to the thousands. In the second case the information regarding the position within the stack, that is, the sixth position was preserved but the information relative to the stack itself was altered, that is from the hundreds to the tens. In a study by Seron and Deloche (1983) using a similar task, patients with Broca’s or Wernicke’s aphasia made both ‘Stack’ and ‘Stack Position’ errors with both groups making more stack than stack position errors. A stack position error occurs where there is erroneous information processing in relation to the position in the stack but with preservation of stack information. Also, both of these error types were found with a similar task in an investigation by Seron and Deloche (1984) involving a mixed group of adult aphasics.

Transcoding Written Verbal Numerals to Spoken Verbal Numerals. All items were completed correctly.

Transcoding Written Verbal Numerals to Arabic Numerals. A. was incorrect on one test item as he wrote 417 instead of 475 when presented with it in written verbal form as ‘four hundred and seventy-five’. In this reading task A. appeared to have read incorrectly seventy as seventeen and then omitted the five, so two types of error were made apparently by him. A. made these errors in a task where he had to read number words and then write them in Arabic notation which is a task described by Deloche and Seron (1982a) involving adult aphasics in which the first type of error made by A. would have been termed a ‘Stack’ error and the
second error type would have been categorized by the term ‘Partial Processing of
the Numeral’. In relation to the stack concept in neuropsycholinguistics, a stack
is a file which contains elements in a serial order such as the names for num-
bers or for the days of the week (Deloche and Seron, 1984). As indicated earlier
when considering numerical processing, a stack error occurs when the stack of a
lexical element of a numeral is incorrectly coded but there is preservation of
information in relation to its position in the stack. For partial processing one of
the elements is not processed and this might occur at the beginning, the middle
or at the end of the presented item.

Additional Item: Tactile Recognition and Naming of Numerals. A. performed
well on the task concerning the tactile recognition and naming of numerals as he
responded quickly and correctly to all items on his first attempts.

CALCULATION TASKS

Operation Symbol and Word Comprehension

Operation Symbol Comprehension. All items were completed correctly.

Operation Word Comprehension. A. showed confusion on several items.
On the problems involving only units A. said that ‘Divided by’ and ‘Plus’
corresponded to the operation name ‘Subtraction’. ‘Times’ did not but when
given ‘Minus’ he was sure that this was the correct operation word. On the prob-
lems involving tens and units A. said that ‘Times’ corresponded and ‘Minus’ did
not correspond to the operation name ‘Subtraction’. Also, ‘Divided by’ corre-
sponded but ‘Times’ did not correspond to the operation name ‘Multiplication’.

Written Arithmetic Tasks

A. made errors on basic subtraction questions and when presented with those
that involved borrowing he added the numbers. Also, he responded to some mul-
tiplication questions as problems in addition. He could give correct answers only
to the very simple multiplication and division questions.

Additional Item: Specific Rectangular and Non-Rectangular Addition
Exercises. A. completed accurately the specific rectangular and non-rectangular
addition exercises designed by the present author and he indicated an understand-
ing of the associated concepts.

Oral Arithmetic Tasks

A. was correct on the addition and subtraction questions used as oral arith-
metic tasks. He corrected himself quickly after giving two erroneous answers to
multiplication exercises. A. was unable to complete any division problems pre-
sented orally. Also, as indicated above A. had experienced some difficulties on
the Arithmetic subtest of the WISC-R (UK) which involves an oral presentation
and response although more verbal interpretation is involved than on the simple
arithmetic tasks mentioned in this section.

Additional Item: Recitation of Multiplication Tables. A. showed very marked
difficulties in relation to remembering the multiplication tables. In an attempt to
recite them he tended to work out each answer by counting on from the previous response. Hence he was able to reply correctly on some items. Although he used this method frequently he was able to respond accurately sometimes using direct recall when smaller numbers were involved, for example, $2 \times 3 = 6$. Also, he was able to retrieve directly most of the items from the 10 and 11 times tables. Some of the inaccurate responses that A. made in the multiplication tables were termed ‘Bond’ errors and ‘Shift’ errors by Temple (1991, 1997b). ‘Bond’ errors occur when an individual retrieves an answer which indicates that the correct table is being accessed for one of the numbers in the computation but the selection is incorrect, for example, $6 \times 3 = 15(18)$. Hence, there was the consideration of table entries being stored in an interconnected fashion and bond errors being categorized as semantic (Temple, 1997b). ‘Shift’ errors are described when erroneous responses are not alternative table values and they contain a single digit that is incorrect, for example, $9 \times 3 = 37(27)$. The second digit is correct and the answer does not occur in the 9 or the 3 times tables. Also, A. made ‘Perseverative’ errors as described by Temple and Marriott (1998). These are inaccurate responses that have occurred previously in relation to other problems.

CHILD M.

M. was aged 10 years 6 months when he participated initially in the project and he was in a mainstream setting which had been the case for all of his primary education. Also, he had not received specialist educational support in any form before or during his school years. As indicated above, M. was involved with various assessments but mainly information is discussed here concerning the Dyscalculia Test Battery. However, a few aspects involving specific numerical tasks are mentioned in relation to this. On the WISC-R (UK) he did not perform well on the Arithmetic subtest. On Digit Span he showed some facility on the Forward Digit Span task which is a relatively simple short-term memory test. This is in contrast to his particularly weak performance on Backward Digit Span which as highlighted earlier is a more complex task. In fact he could repeat correctly only two numbers backwards for both trials on the first item in this section. Hence, his difficulties might be considered in terms of problems associated with aspects of working memory involving the manipulation of stored numerical data.

DYSCALCULIA TEST BATTERY ASSESSMENT

NUMERAL PROCESSING TASKS

Magnitude Comparison

Arabic Magnitude Comparison. All items were completed correctly.
Spoken Verbal Magnitude Comparison. All items were completed correctly.
Written Verbal Magnitude Comparison. All items were completed correctly.
Transcoding Tasks

*Transcoding Arabic Numerals to Spoken Verbal Numerals.* M. was correct on all items. As noted earlier, this test was similar to the reading task used in the study described by Power and Dal Martello (1997) involving 7-year-old Italian children.

*Transcoding Spoken Verbal Numerals to Written Verbal Numerals.* M. made one error when he gave a written verbal response of ‘one thousand two hundred and thirty-eight’ instead of ‘one thousand two hundred and seventy-eight’ which was the correct response. This lexical error was termed a ‘Substitution’ error by Noël and Seron (1995) in relation to a neuropsychological investigation of an adult with a suggested diagnosis subsequently of Alzheimer’s disease. M. was able to correct himself on a second attempt.

*Transcoding Spoken Verbal Numerals to Arabic Numerals.* M. gave several erroneous answers when he was required to write Arabic numerals in response to spoken verbal numerals in a dictation task similar to that described in the study highlighted earlier by Power and Dal Martello (1990) involving 6–8-year-old Italian children. M.’s errors varied in type and in relation to the ease with which he was able to correct them himself. He made two syntactical errors where extra zeros were inserted, namely 10,016 instead of 116 but he was able to correct himself on a second attempt and in another case when he wrote 30,264 instead of 3,264. However, for this item he was incorrect more dramatically on a second attempt when he wrote an erroneous answer as 3,000 200 64 but 2 weeks later he was able to correct himself. Both of his responses are examples of a child resorting to concatenation as opposed to applying the operation termed ‘over-writing’ by Power and Dal Martello (1990).

In a study by Seron and Fayol (1994), a similar task was presented to 7-year-old children from France and Wallonia, a region of Belgium. Some similar errors were made by these children and the investigators applied the terms ‘Full Literal Transcoding’ errors to those such as 10,016 instead of 116 and ‘Partial Literal Transcoding’ errors to those such as 30,264 instead of 3,264 in relation to M.’s errors mentioned above. It is interesting to note here also, that on a second attempt M. made a full literal transcoding error when he wrote 3,000 200 64 instead of 3,264. Also, M. made another syntactical error but of a different type in which he reversed the order of two digits in a four-digit number, when he wrote 5,063 instead of 5,603 and in fact, he made the same error on a second attempt but 2 weeks later he corrected himself.

M. made one lexical error when he wrote 7,822 instead of 7,802 but he gave the correct answer on a second attempt. In the discussion concerning child A. reference was made to a case study described by Temple (1989, 1997a, b) concerning an 11-year-old boy. An impairment in lexical processing was demonstrated which resulted in the incorrect selection of digits when he was required to read Arabic numerals or to write Arabic numerals to dictation whereas there was accurate syntactic processing. Also, he made more errors when reading numeral words than when reading Arabic numerals but the errors were of a similar form.
As noted above, M. did make one lexical error when writing Arabic numerals to dictation but he was accurate on items in the tasks where he was required to read Arabic numerals or number words. All of the errors made by M. in the dictation task where he had to respond by writing Arabic numerals occurred toward the end of a session and he was incorrect on a second attempt for two of the test items which were corrected by him in a further session 2 weeks later. Hence, there is an indication perhaps of the amount of effort that some children have to make in order to develop their numerical skills.

Various error types that were demonstrated by M. and others have been highlighted in neuropsychological data from adult patients as discussed, for example, by Deloche and Seron (1982a, b, 1987), Seron and Deloche (1983, 1984) and McCloskey and Caramazza (1987). Specifically in this context is an interesting case study reported by Cipolotti et al. (1994) of an adult neurological patient with a non-fluent aphasia and right arm weakness after his stroke involving the left parietal lobe. When he was seen for a neuropsychological assessment there was almost complete recovery in terms of his motor impairment and aphasia. A neuropsychological deficit was demonstrated concerning a very weak performance on the Arithmetic subtest on the WAIS and on the Graded Difficulty Arithmetic Test (Jackson & Warrington, 1986). There was further investigation of his numerical processing and arithmetical skills and the results demonstrated unimpaired numeral reading and comprehension but a transient and selective syntactic impairment in numeral writing to dictated numerals. These errors were discussed in the context of a dissociation between the concatenation and the over-writing rules described in the theoretical formulation by Power and Dal Martello (1990) considered earlier. In line with the children in this study, the adult patient demonstrated syntactical errors involving too many zeros but whereas the children showed errors of this type for numbers above 100, the adult patient showed the errors for numerals with four or more digits on the first day and five or more digits on the second day.

Reference was made to research discussed by Seron et al. (1992) involving 7- and 8-year-old pupils on tasks where they had to read aloud Arabic numerals and to write Arabic numerals after dictation. The children became competent with the over-writing rules with three-digit numbers before they reached the same level of competence with four-digit numbers. Hence, it seemed that the adult man had difficulty with those numerals for which there would be a later application of the over-writing rules and for which there would be the requirement of more effort. Also, after he noticed his errors in writing Arabic numerals, he was able to write correctly all of the items. As indicated above, the boy M. who participated in my study inserted extra zeros in a three-digit number and a four-digit number. He was able to correct himself quite easily on the three-digit number where he had inserted two zeros in the hundred position. However, on the four-digit number he made an insertion of one zero in the thousand position on the first attempt and a combination of three and two zeros respectively for the thousand and hundred positions on the second attempt before giving a
correct answer 2 weeks later. Hence, M. demonstrated a very persistent weakness which was reflected in his continued use of concatenation with this higher-order number but eventually with the necessary effort he was able to overcome his difficulty and he could apply appropriately, the over-writing rule.

Transcoding Arabic Numerals to Written Verbal Numerals. Some of M.’s errors are interesting in relation to a case study described by Cohen et al. (1994) of a 43-year-old right-handed man who was severely aphasic after a left hemispheric subdural hemorrhage. His performance on various word and non-word reading tasks indicated deep dyslexia. Specifically, he showed difficulties reading aloud non-words and unfamiliar Arabic numerals but there was a significant improvement in relation to real words and familiar Arabic numerals. In reading unfamiliar numerals the authors suggested that the man appeared to use his semantic route as he showed a tendency to decompose the unfamiliar numerals into meaningful sub-groups of familiar numerals. As an example, when presented with ‘726’ the patient responded with ‘seven, two, six’.

Some errors were made by M. concerning the transcoding of visually presented Arabic numerals to written verbal numerals. For 7,802 M. wrote ‘seven thousand eight hundred zero two’ and for 2,191 M. wrote ‘two thousand one ninety-one’. As suggested by Cohen et al. (1994) in consideration of the type of responses given by the man described above, a complex non-familiar numeral was fractionated into more simple and basic numerals. M. demonstrated an apparently similar process but concerning the transcoding of visually presented Arabic numerals to written verbal numerals as opposed to the transcoding of visually presented Arabic numerals to spoken verbal numerals. The examples described here by Cohen et al. (1994) and by myself in relation to M. were similar also to those described by Power and Dal Martello (1997) as ‘Fragmentation’ errors. In a particular instance, a child read the Arabic numeral 495 and gave a spoken response of ‘forty nine and five’.

Also, when transcoding visually presented Arabic numerals to written verbal numerals M. made a stack error as described by Seron and Deloche (1983) in relation to adults with Broca’s or Wernicke’s aphasia and noted by Seron and Deloche (1984) in a mixed group of adult aphasics, with both investigations involving the use of similar tasks. When M. was presented with 9,034 he responded with ‘nine thousand three hundred and four’ indicating the preservation of the third position within the stack but not in relation to the stack itself, with the hundreds replacing the tens in this example. As in the case of the erroneous responses mentioned earlier, these errors occurred toward the end of the assessment session and after a week he gave correct answers to the first two questions but made the same mistake again for 9,034 although he corrected himself on a further attempt during the same session. Hence, again the specific nature of some errors was emphasized and their persistence as well as the effort required to overcome the difficulties associated with their production.

Transcoding Written Verbal Numerals to Spoken Verbal Numerals. All items were completed correctly.
Transcoding Written Verbal Numerals to Arabic Numerals. M. made two errors which involved the insertion of extra zeros. Specifically, for ‘three thousand two hundred and sixty-four’ he wrote 32,064 and for ‘two thousand three hundred and eleven’ he wrote 23,011. In a study mentioned earlier by Deloche and Seron (1982a) with adult aphasics patients from France and Wallonia in Belgium and involving a similar task, these types of errors were noted and they were classified as ‘Intra-item Perseveration’ errors. Again, these errors had been made by M. toward the end of a session but when re-tested after a 4-week break he wrote in response respectively to these two numbers, 31,264 and 21,311. Hence, M. made two errors again where the length of the numeral was increased and this was due to the insertion of the digit ‘1’, as described for example by Deloche and Seron (1982a). As indicated, he wrote respectively ‘12’ instead of ‘2’ and ‘13’ instead of ‘3’ but when he was questioned about his answers he was able to give correct responses.

Additional Item: Tactile Recognition and Naming of Numerals. M. made several errors when he was asked to find numbers at random and then to name them while they were covered by a cloth after which they were checked for correct identification. He found the task easier when specific numbers were requested and he made only one error in finding a 3 instead of a 5 but a correct number was found on a second attempt. Hence, M. experienced some difficulty involving the tactile modality mainly for number naming more than recognition. However, when he was assessed in a later session in relation to the tactile naming of numbers he was able to perform accurately on this task.

CALCULATION TASKS

Operation Symbol and Word Comprehension

Operation Symbol Comprehension. When the operation name ‘Subtraction’ was presented M. identified incorrectly the division symbol. When presented with the operation name ‘Division’ M. identified correctly the division symbol. Later when presented with the operation name ‘Subtraction’ he identified correctly the symbol for subtraction. These identifications occurred for items involving units, tens and hundreds.

Operation Word Comprehension. When presented with the operation name ‘Subtraction’ M. identified incorrectly ‘Divided by’. When presented with the operation name ‘Division’ M. was correct in identifying ‘Divided by’. These identifications occurred in relation to items involving units, tens and hundreds.

Written Arithmetic Tasks

M. managed some of the simpler exercises involving all four arithmetical operations but there were errors also on some of the very elementary questions. On one exercise he added the numbers when the required operation was subtraction. Also, he demonstrated difficulties with carrying on the more complicated addition exercises and he showed a lack of understanding in relation to borrowing for the more advanced subtraction problems.
Additional Item: Specific Rectangular and Non-Rectangular Addition Exercises. M. demonstrated a particular difficulty with the specific non-rectangular addition problems mentioned in the discussion on child A. He made errors of the first specific type shown by some other children in studies by Friend (1979) and Lawson (1986, 1989) as he added a number from another column on to the single digit in the far left column. M. did not appear to have understood conceptual issues concerning the join of a single set and the sum of a single number. However, he seemed to develop more understanding of these concepts and to answer related questions correctly after a relevant discussion and instruction with a simple technique presented in one teaching session and described by Lawson (1990, 1995a, 2000a, 2001a). When re-assessed he completed accurately all of the non-rectangular exercises.

Oral Arithmetic Tasks

M. made one error in each group of questions concerning addition, subtraction and multiplication but he was able to correct himself very quickly. He could not complete any of the division exercises presented in an oral form. Also, as indicated above his performance was not strong on the Arithmetic subtest of the WISC-R (UK) involving aurally presented word problems requiring oral responses.

Additional Item: Recitation of Multiplication Tables. M. demonstrated particular problems in remembering the multiplication tables. He was able to recall the answers sometimes when smaller numbers were involved and many items from the 2, 5 and 10 times tables. When M. attempted to recite the tables, if he could not retrieve an answer directly he tended to work out each item by counting on from the previous answer. He made some ‘Bond’, ‘Shift’ and ‘Perseverative’ errors that were noted earlier in the discussion concerning child A.

CHILD L.

Initially when L. participated in the program he was aged 8 years 5 months. He attended a mainstream primary school and prior to the interventions he had not received specialist support in any form. His presentation was characteristic of the autism spectrum, more precisely Asperger syndrome although he was making noticeable progress in coping with specific problematic aspects. Some relatively mild areas of concern were apparent in relation to particular tasks involving fine or gross motor skills associated respectively, with graphic and certain physical activities. Reference could be made to Dziuk et al. (2007) for a recent investigation and discussion involving basic motor coordination and dyspraxic aspects concerning impairments in the performance of skilled gestures in relation to individuals on the autism spectrum. Generally L. was doing well in terms of his academic work at school but he had experienced considerable difficulty in learning the multiplication tables.
As mentioned earlier, a more comprehensive discussion including the assessments and interventions concerning aspects of L.’s disposition including his social and communication difficulties as well as those relating to the multiplication tables had been given in a presentation by Lawson (2005). In relation to the considerable controversy concerning diagnostic criteria I am sympathetic to Wing (2005) who favors a multidimensional as opposed to categorical approach to the study of the autism spectrum. In line with this perspective I tried to clarify and facilitate development in L.’s weaker areas with the support of his relative strengths in other aspects.

Only L.’s problems associated with learning the multiplication tables and the assessment and intervention are highlighted here. However, a few points will be mentioned in relation to these aspects as for the other two children in this presentation. Also, as indicated earlier this boy participated in the program more recently so he was administered some different or updated versions of tests. In particular, he was assessed on the Arithmetic subtest of the WISC-III UK, Weschler (1992) and this was not specifically problematic for him. Also, L. was presented with the Recall of Digits, Forward and Backward subtests on the British Ability Scales II (BAS II), Elliot et al. (1996). The digits on these subtests are presented at a faster rate in comparison with similar tasks on the WISC-R. L. did not demonstrate difficulties particularly with either section so within this context it was not suggested that the storage and manipulation of numerical information was problematic.

### Dyscalculia Test Battery Assessment

#### Numeral Processing Tasks

**Magnitude Comparison**

*Arabic Magnitude Comparison.* All items were completed correctly.

*Spoken Verbal Magnitude Comparison.* All items were completed correctly.

*Written Verbal Magnitude Comparison.* All items were completed correctly.

**Transcoding Tasks**

*Transcoding Arabic Numerals to Spoken Verbal Numerals.* All items were completed correctly.

*Transcoding Spoken Verbal Numerals to Written Verbal Numerals.* All items were completed correctly.

*Transcoding Spoken Verbal Numerals to Arabic Numerals.* All items were completed correctly.

*Transcoding Arabic Numerals to Written Verbal Numerals.* All items were completed correctly.

*Transcoding Written Verbal Numerals to Spoken Verbal Numerals.* All items were completed correctly.
Transcoding Written Verbal Numerals to Arabic Numerals. All items were completed correctly.

Additional Item: Tactile Recognition and Naming of Numerals. L. found both tactile tasks quite demanding and he responded slowly to each request. However, without any time constraints when he had to retrieve named numbers, he was incorrect on only one item. For the number ‘2’ his response was ‘8’ but he was correct on a second attempt. When he was asked to find individual numbers at random and to name them one at a time he was incorrect on three items. Specifically, he responded with ‘3’ for a ‘2’, ‘8’ for a ‘3’ and ‘6’ for an ‘8’ but he gave correct responses on second attempts in each case.

CALCULATION TASKS

Operation Symbol and Word Comprehension

Operation Symbol Comprehension. All items were completed correctly.

Operation Word Comprehension. All items were completed correctly.

Written Arithmetic Tasks

These tasks were used to investigate the retrieval of arithmetic facts and execution of the calculation procedures for addition, subtraction, multiplication and division. As indicated earlier a standardized arithmetic test, the WOND was administered with the NO section requiring written answers.

The test results showed that L. could manage some arithmetic questions involving addition and subtraction. However, he added instead of subtracting for some questions whether presented horizontally or vertically. Some of his errors indicated difficulties with carrying and borrowing. Also, he was correct on a very simple multiplication question but he could not attempt any division exercises. In relation to the multiplication exercises he made a ‘Shift’ error where one digit in the answer is incorrect and the erroneous response is not a table entry as noted earlier. On another question he added instead of multiplying.

Additional Item: Specific Rectangular and Non-Rectangular Addition Exercises. L. did not demonstrate any difficulties with the exercises in groups (A1, B1) so he was not administered any other questions.

Oral Arithmetic Tasks

These tasks probed arithmetic fact retrieval and involved the four calculation procedures. Each problem was presented aurally and the child said the answer aloud. Five questions were presented with each item involving two numbers in the range 0–9 for the arithmetic operations addition, subtraction, multiplication and division.

L. made one error when he responded $7 \div 7 = 56(49)$. However, soon he corrected himself and gave the answer 1. He noted that he had multiplied instead of divided but he had given an incorrect table entry which was an adjacent answer in the same table, that is, a mistake termed a ‘Bond’ error as indicated above.
Also, as mentioned earlier L. was administered the Arithmetic subtest on the WISC-III UK. He was presented aurally with questions that required only an oral response. He did not have to read any items. L. responded accurately until he could not give any more answers.

**Additional Item: Recitation of Multiplication Tables.** The child was asked to recite any of the multiplication tables or any items that he/she was able to recall. The results for the responses given by L. are presented below.

<table>
<thead>
<tr>
<th>Table</th>
<th>Incorrect Items (Correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$9 \times 2 = 19(18)$</td>
</tr>
<tr>
<td>3</td>
<td>$5 \times 3 = 16(15)$</td>
</tr>
<tr>
<td></td>
<td>$6,7,8,9$</td>
</tr>
<tr>
<td>4</td>
<td>$6,7,8,9$</td>
</tr>
<tr>
<td>5</td>
<td>Correct responses on all items</td>
</tr>
<tr>
<td>6</td>
<td>$3 \times 6 = 19(18)$</td>
</tr>
<tr>
<td>7</td>
<td>$2,3,4,5,6,7,8,9,12$</td>
</tr>
<tr>
<td>8</td>
<td>$2,3,4,5,6,7,8,9,12$</td>
</tr>
<tr>
<td>9</td>
<td>$8 \times 9 = 62(72)$</td>
</tr>
<tr>
<td>10</td>
<td>Correct responses on all items</td>
</tr>
<tr>
<td>11</td>
<td>$10 \times 11 = 111(110)$</td>
</tr>
<tr>
<td>12</td>
<td>$2,3,4,5,6,7,8,9,11,12$</td>
</tr>
</tbody>
</table>

**Recall of Items from the Multiplication Tables**

As expected, when assessed initially L. showed some difficulties in relation to remembering the multiplication tables. If he had difficulty with recall he might work out an answer by counting on from the previous response. Hence he was able to reply correctly for some items. As shown above, for the initial assessment L. gave correct replies, no responses or incorrect answers involving ‘Shift’ errors. During the remedial sessions he made ‘Bond’, ‘Shift’ and ‘Perseverative’ errors.

**GENERAL DISCUSSION AND CONCLUSION**

The children presented here demonstrated some difficulties in all three areas concerning numerical processing and knowledge of number facts as well as procedures. In the studies discussed by Temple (1989, 1991, 1994a, b, 1997a, b), selective deficiencies were highlighted in relation to lexical number processing, number fact and procedural disorders. It was suggested that these different types of selective impairment in developmental dyscalculia indicated a modular organization of the developing arithmetical system. The perspective considered by Temple in relation to this system allowed for some communication between modules.
More recently Kaufmann (2002) described an adolescent young man who was 14 years of age initially when he participated in their investigation. His presentation included severe developmental dyscalculia and problems with literacy including reading and writing as well as marked difficulties with spelling. He had relatively preserved procedural skills and marked deficits concerning number fact retrieval with particularly problematic areas relating to multiplication and division. Also, he performed poorly on Forward and Backward Digit Span tasks. As indicated above in my presentation here, Child A. experienced difficulties on Forward and Backward Digit Span tasks and Child M. had problems specifically associated with Backward Digit Span. As noted earlier, the interpretations relating to their performances in this respect were considered in terms of impairments concerning working memory involving numerical information. In the Kaufmann (2002) study it was suggested that the memory deficit demonstrated by the young man was not just the result of poor numeric fact representations in long-term memory but there was a substantial contribution from a deficiency specifically of a numerical nature relating to working memory. In a later investigation concerning the same young man when he was 18 years of age, Kaufmann et al. (2004) emphasized the significance of working memory in the activation of information from long-term memory. Numerical fact retrieval difficulties were considered in relation to a combined access and storage deficiency.

As indicated earlier in the context of my own research, Child L. did not present with difficulties particularly concerning the Forward and Backward Digit Span subtests. Also as noted above, the presentation rate for digits on these subtests was faster than the rate on similar tasks used with the other two participants. Facilitation of working memory and its associations with long-term memory in relation to numerical development was promoted via the implementation of a specific multisensory technique within the context of a remedial approach involving the integration of factual, procedural and conceptual aspects of numerical knowledge.

In my research presented here, the children had been highlighted by their teachers or parents because they had demonstrated a particular difficulty in learning the multiplication tables. Other areas of numerical difficulty were clarified with the assistance of the dyscalculia test battery as well as some strengths. In the context of a flexible and interconnected system, it should be possible to use strengths in various areas to compensate for weaknesses elsewhere or to promote development in weaker areas and to facilitate the maturation of numerical concepts and skills. The importance of associations and commonalities as well as dissociations and differences in relation to specific syndromes has been highlighted in the neuroconstructivist approach (Karmiloff-Smith, 1998; Oliver et al., 2000) with increasingly specialized or modularized cortical pathways being produced by development. The teaching/learning approach suggested and implemented by the present author (Lawson, 1995b, 2000b, 2001a) was aimed at using the various strengths of the children to influence their developmental courses in a positive direction with the assistance of a specific multisensory technique.
As indicated above, this involved the visual, auditory, tactile and kinesthetic modalities in relation to items in the multiplication tables. Also, a slightly modified approach was applied to assist the apprehension of arithmetical operation symbols, words and names but with the operation items being changed in the process as opposed to the operands.

The discussion presented here concerned information that had been obtained from an assessment involving a dyscalculia test battery. The aim of the project was to analyze the children’s numerical difficulties and to provide a remedial approach that would help them to learn the multiplication tables. Also, the facilitation of more mature development was expected in relation to number concepts and numerical skills through the establishment of flexible associations between specialized areas or modules as emphasized by Dehaene (1997). It is apparent from the results obtained that a detailed assessment with the aid of a dyscalculia test battery provided clear information concerning the numerical difficulties experienced by the participants. After taking part in the remedial sessions, the children could access any item from the multiplication tables, obtain the answers more quickly and retrieve them more often by direct recall. Also, they could use other approaches such as locating an item near the correct answer as a cue and then recall the required item or count up or down in various ways to reach the appropriate answer. Alternatively, they might use some arithmetical knowledge involving addition or subtraction, perhaps that associated with doubles or make use of commutativity as an aid to retrieving the correct answer. Also, the three children discussed here noticed specific patterns in the 5, 10 and 11 times tables and the use of this knowledge was encouraged during the sessions to assist the retrieval of other items.

In a particular study by Krueger (1986) it was indicated that adults could use odd–even rules efficiently, for example, if either multiplier is even the answer must be even, in a product verification task. This was an aspect that I emphasized when teaching the multiplication tables and the children were encouraged to use this information to help them check and correct answers that they had retrieved. As the sessions progressed, use of this knowledge was demonstrated very clearly to some extent as it was used overtly. Hence, in addition to some strategies mentioned already the use of another mature strategy demonstrated in adults had been developed and was being used appropriately by these children.

Sherin and Fuson (2005) categorize multiplication strategies in relation to what they term as the number-specific computational resources that are used in their execution. The count-all strategy which involves counting from ‘1’ to the product is available to most individuals when they become involved with multiplication. Also, because of their experiences concerning addition the children have access to related strategies which these authors call additive calculations. When considering other categories, count-by strategies were discussed as pupils learn count-by sequences of numbers such as ‘4, 8, 12, 16, . . .’. Pattern-based strategies which might involve for example, 0’s, 1’s or 10’s are learnt often alongside count-by sequences. The learned product strategy involves many
number-specific resources, namely the multiplication triads. Also, hybrid strategies might be used which involve various combinations of those noted above. As indicated earlier, the young people involved with the approach discussed here used various strategies and combinations which could be categorized in this way. Also, they could recognize a variety of applications as well as implement appropriately their newly acquired understanding and skills in different situations.

As suggested by Thelen and Smith (1994) while acknowledging the multimodal nature of most of our experiences of objects and events, when considering perception and action in the real world, development might be concerned more with the selection than with the construction of the most relevant multimodal associations. The participants in my research discussed here had considerable difficulties in accessing items from the multiplication tables and initially construction of some numerical ideas was not easy. The aim of the teaching/learning approach used here was to help the children with the selection and integration of the most salient information from the stimuli which were presented simultaneously in various modalities. As they developed the ability to store and access sufficient quantities of information and to form meaningful associations, newly acquired knowledge could be used in the construction of further numerical concepts and ideas given different applications. Hence, their numerical and certain other aspects of their mathematical development were enhanced by the careful analysis using the dyscalculia test battery and the multisensory remedial intervention presented here.

Generally the young people demonstrated substantial improvements in relation to their abilities to access items from the multiplication tables and in the development of their number concepts and arithmetical skills. Boys were selected here but the same approaches to assessment and remediation could be offered to girls. Also, the difficulties demonstrated by the boys were related to the findings in other studies concerning younger children, individuals in a developmental neuropsychological context and adults with acquired neuropsychological disabilities. The successful remedial approach which was implemented after the assessment might be considered applicable to individuals with similar numerical difficulties but generally of a different disposition. In fact, L. was much younger and as indicated above his overall presentation was quite different when compared with the other two boys. Apart from the recall of more items and the development of many strategies for accessing items from the multiplication tables, these children were able to apply their knowledge in various contexts, hence demonstrating their enhanced numerical understanding and ability to generalize.

ACKNOWLEDGMENTS

My grateful thanks to the children and their families as well as school staff and other supportive individuals who contributed to the investigations presented here.
REFERENCES


