HET AMSTERDAMS LYCEUM

Naam: Hg : Rijen en Reeksen Klas: V5

Vak: wis A & wis C Datum:

Cijfer:

① a) 48, 51, 54, 57, 60, 63, ...

$\underbrace{}_{+3}$ $\underbrace{}_{+3}$ $\underbrace{}_{+3}$ etc → constant verschil, dus RR

recursieve formule : $u_n = u_{n-1} + 3$ met $u_0 = 48$

directe formule : $u_n = 48 + 3n$

b) 20, 10, 5 ; 2,5 ; 1,25 ; 0,625; ...

$\underbrace{}_{\cdot\frac{1}{2}}$ $\underbrace{}_{\cdot\frac{1}{2}}$ $\underbrace{}_{\cdot\frac{1}{2}}$ etc → constant quotiënt, dus MR

recursieve formule : $u_n = \frac{1}{2} \cdot u_{n-1}$ met $u_0 = 20$

directe formule : $u_n = 20 \cdot (\frac{1}{2})^n$

② 1 jan. 2006 : € 1000,-) +4% − € 100,- ans = 1000 [ENTER]

1 jan. 2007 : € 940,- ans · 1,04 − 100 [ENTER]

1 jan 2008 : € 877,60

⋮

1 jan 2018 : € 98,45

1 jan 2019 : € 2,39

1 jan 2020 : € −97,51 ← dus op 1 jan 2020 is saldo onvoldoende.

③ a) $u_n = n^3 - 3n + 1$

8e term, dus $u_7 = 7^3 - 3 \cdot 7 + 1 = 323$

b) $u_n = 5 + 2\sqrt{u_{n-1}}$ met $u_0 = 100$ ans = 100 [ENTER]

8e term, $u_7 \approx 11{,}90$ (via GR) $5 + 2\sqrt{ans}$ [ENTER]

④ $u_n = u_{n-1} + 6$ met $u_0 = 2$

u_n : 2, 8, 14, 20, 26, 32, ...

$\underbrace{}_{+6}$ $\underbrace{}_{+6}$ $\underbrace{}_{+6}$ etc → constant verschil, dus RR

directe formule : $u_n = 2 + 6n$, dus $a = 6$.

⑤ a) $u_n = 2n+7$, $u_0 = 7$, $u_5 = 2\cdot5+7=17$

$\sum_{k=0}^{5} u_k \overset{RR}{=} \frac{1}{2}(5+1)(7+17) = 72$

b) $v_n = n^2+3$

$\sum_{k=0}^{4} v_k = v_0 + v_1 + v_2 + v_3 + v_4$

$= 3 + 4 + 7 + 12 + 19$

$= 45$

c) $w_n = 2^n$

$\sum_{k=0}^{4} w_k \overset{MR}{=} \frac{1\cdot(1-2^{4+1})}{1-2} = \frac{1-32}{-1} = 31$

d) $z_n = 2u_{n-1} + n$ met $z_0 = 2$

$\sum_{k=0}^{4} z_k = z_0 + z_1 + z_2 + z_3 + z_4$

$= 2 + 5 + 12 + 27 + 58$

$= 104$

$z_0 = 2$
$z_1 = 2\cdot2+1=5$
$z_2 = 2\cdot5+2=12$
$z_3 = 2\cdot12+3=27$
$z_4 = 2\cdot27+4=58$

⑥ a) $17+21+25+\dots+149 =$

$\sum_{k=0}^{33} u_k \overset{RR}{=} \frac{1}{2}(33+1)(17+149)$

$= 2822$

($u_0 = 17$, steeds $+4 \rightarrow$ RR)

$u_n = 17+4n$
$149 = 17+4n$
$132 = 4n$
$n = 33$
$u_{33} = 149$

b) $89+83+77+\dots+17 =$

$\sum_{k=0}^{12} u_k \overset{RR}{=} \frac{1}{2}(12+1)(89+17)$

$= 689$

$u_0 = 89$, steeds $-6 \rightarrow$ RR
$u_n = 89-6n$
$17 = 89-6n$
$-72 = -6n$
$n = 12$
$u_{12} = 17$

⑦ a) $u_n = 7n+1$

$\sum_{k=0}^{n} u_k = \frac{1}{2}(n+1)(1+7n+1) = \frac{1}{2}(n+1)(2+7n) = \frac{1}{2}(7n^2+9n+2)$

$= 3\frac{1}{2}n^2 + 4\frac{1}{2}n + 1$

b) $\left.\begin{array}{l} y_1 = 3\frac{1}{2}x^2+4\frac{1}{2}x+1 \\ y_2 = 1000 \end{array}\right\}$ optie table geeft: $x = 16 \rightarrow y_2 = 969$
$x = 17 \rightarrow y_2 = 1089$

Dus vanaf $\underline{n = 17}$

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(8) $12, 16, 20, 24, 28, \ldots$ steeds $+4 \rightarrow$ RR

$u_n = 12 + 4n$

$\sum_{k=0}^{n} u_k \overset{RR}{=} \frac{1}{2}(n+1)(12 + 12 + 4n) = \frac{1}{2}(n+1)(24+4n)$

$= (n+1)(12+2n)$

$= 2n^2 + 14n + 12$

(9) $\sum_{k=0}^{n} (3k) \overset{RR}{=} \frac{1}{2}(n+1)(0 + 3n) = \frac{1}{2}(n+1)\cdot 3n = 1\frac{1}{2}n^2 + 1\frac{1}{2}n$

(10) a) $\sum_{k=0}^{15} 100 \cdot 1{,}1^k \overset{MR}{=} \dfrac{100(1-1{,}1^{15+1})}{1-1{,}1} \approx 3594{,}97$

b) $v_n = 200 \cdot 0{,}98^n$

$\sum_{k=0}^{14} v_k \overset{MR}{=} \dfrac{200(1-0{,}98^{14+1})}{1-0{,}98} \approx 2614{,}31$

(10) c) $w_n = 1{,}45 \cdot w_{n-1}$ met $w_0 = 50$

$w_n = 50 \cdot 1{,}45^n$

$\sum_{k=0}^{12} w_k = \dfrac{50(1-1{,}45^{12+1})}{1-1{,}45} \approx 13805{,}76$

(11) a) $u_n = 10000 \cdot 0{,}6^n$

$\sum_{k=0}^{n} u_k = \dfrac{10000(1-0{,}6^{n+1})}{1-0{,}6} = \dfrac{10000(1-0{,}6^n \cdot 0{,}6^1)}{0{,}4} = 25000(1-0{,}6 \cdot 0{,}6^n)$

$= 25000 - 25000 \cdot 0{,}6 \cdot 0{,}6^n$

$= 25000 - 15000 \cdot 0{,}6^n$

b) $y_1 = 25000 - 15000 \cdot 0{,}6^x$
$y_2 = 24999$

(optie intersect geeft $x \approx 18{,}8$
Dus $\underline{x = 19}$)

PS. eigenlijk optie table gebruiken ~~maar~~ maar is lastiger te interpreteren in dit geval.
$x = 18 \rightarrow y \approx 24998$
$x = 19 \rightarrow y \approx 24999$

(12) a) $u_n = 18 \cdot 0{,}4^n$

$$\sum_{k=0}^{n} u_k \overset{mR}{=} \frac{18(1-0{,}4^{n+1})}{1-0{,}4} = \frac{18(1-0{,}4^n \cdot 0{,}4^1)}{0{,}6} = 30(1-0{,}4 \cdot 0{,}4^n)$$

$$= 30 - 30 \cdot 0{,}4 \cdot 0{,}4^n$$

$$= 30 - 12 \cdot 0{,}4^n$$

b) $\left.\begin{array}{l} y_1 = 30 - 12 \cdot 0{,}4^x \\ y_2 = 29{,}99 \end{array}\right\}$ optie table geeft: $x = 7 \rightarrow y = 29{,}98$

$x = 8 \rightarrow y = 29{,}992$.

Dus vanaf $n = 8$.

(13) $u_n = u_{n-2} + 3u_{n-1}$ met $u_0 = 2$ en $u_1 = 5$

$u_2 = u_0 + 3u_1$ (overal $n=2$ invullen)

$u_2 = 2 + 3 \cdot 5 = 17$

$u_3 = 5 + 3 \cdot 17 = 56$

$u_4 = 17 + 3 \cdot 56 = 185$

$u_5 = 56 + 3 \cdot 185 = 611$

$u_6 = 185 + 3 \cdot 611 = 2018$

$u_7 = 611 + 3 \cdot 2018 = 6665$

$u_8 = 2018 + 3 \cdot 6665 = 22013$

(14) $u_{n+2} = u_n^2 + u_{n+1}$ met $u_0 = 2$ en $u_1 = 5$

$u_2 = u_0^2 + u_1$ (overal $n=0$ invullen)

$u_2 = 2^2 + 5 = 9$

$u_3 = 5^2 + 9 = 34$

$u_4 = 9^2 + 34 = 115$

$u_5 = 34^2 + 115 = 1271$

$u_6 = 115^2 + 1271 = 14496$

$u_7 = 1271^2 + 14496 = 1629937$

$u_8 = 14496^2 + 1629937 = 211763953$

(15) a) $u_n = 11{,}3 \text{ mjd} \cdot 1{,}074^n$

b) $$\sum_{k=0}^{12} u_k \overset{mR}{=} \frac{11{,}3 \text{ mjd}\,(1-1{,}074^{12+1})}{1-1{,}074} \approx 233{,}58 \text{ mjd dollar}$$

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Vak: wis A & wis c Datum:

(16)

$u_0 = 18 \; \big) +5,2$

$u_1 = 23,2 \; \big) +5,2 \cdot 0,8$

$u_2 = 27,36$

$\vdots$

$u_n = 5,2 \cdot 0,8^n$

a) groei in 8e week: $u_7 = 5,2 \cdot 0,8^7 \approx 1,1$ cm

b) $\sum_{k=0}^{7} u_k = \dfrac{5,2(1-0,8^{7+1})}{1-0,8} \approx 21,6$ cm

c) $\sum_{k=0}^{9} u_k = \dfrac{5,2(1-0,8^{9+1})}{1-0,8} \approx 23,2$ cm

Dus hoogte is dan $18 + 23,2 = 41,2$ cm

(17)

$\underbrace{4,9 \; ; \; 14,7}_{+9,8} \; ; \; \underbrace{\; ; \; 24,5}_{+9,8}$ etc $\rightarrow$ constant verschil van $+9,8 \rightarrow$ RR

$u_n = 4,9 + 9,8n$

$$\sum_{k=0}^{n} u_k \overset{RR}{=} \tfrac{1}{2}(n+1)(4,9 + 4,9 + 9,8n)$$
$$= \tfrac{1}{2}(n+1)(9,8 + 9,8n)$$
$$= (n+1)(4,9 + 4,9n)$$
$$= 4,9n^2 + 9,8n + 4,9$$

Dus $a = 4,9$
$b = 9,8$
$c = 4,9$

(18) a) $\sum_{k=2}^{9} (5k+6) \overset{RR}{=} \tfrac{1}{2}(9+1-2)(16+51)$

$= \tfrac{1}{2} \cdot 8 \cdot 67$

$= 268$

wegens ontbreken $k=0$ en $k=1$

$u_2 = 5 \cdot 2 + 6 = 16$
$u_9 = 5 \cdot 9 + 6 = 51$

u_2 is eerste term.

b) $\sum_{k=2}^{9} (3 \cdot 2^k) \overset{MR}{=} \dfrac{12 \cdot (1-2^{9+1-2})}{1-2}$

$= \dfrac{12 \cdot (1-2^8)}{-1}$

$= 3060$

wis A

↓

(19) a) $y = a + b\sin(cx)$, periode $= \frac{2\pi}{1} = 2\pi$; evenwichtsstand bij $y=0$
amplitude $= 1$

b) periode $= \frac{2\pi}{3} = \frac{2}{3}\pi$, evenwichtsstand bij $y=2$, amplitude $=1$

c) periode $= \frac{2\pi}{1/4} = 8\pi$, evenwichtsstand bij $y=3$, amplitude $= 6$

d) periode $= \frac{2\pi}{0{,}1\pi} = 20$, evenwichtsstand bij $y=1$, amplitude $=4$

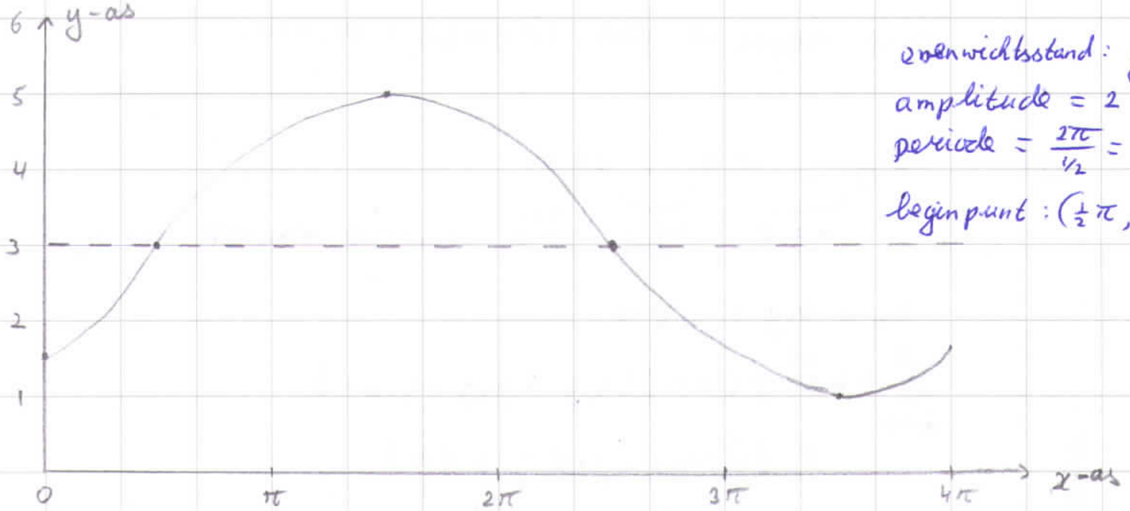
(20) a) $(0,0)$ of steeds een periode verder $(2\pi, 0)$, $(4\pi, 0)$ etc

b) stel $\frac{2}{3}(x-\pi) = 0 \Rightarrow x - \pi = 0 \Rightarrow x = \pi$, dus $(\pi, 4)$ of $(4\pi, 4)$, $(7\pi, 4)$ etc

c) stel $\frac{2}{3}x - \pi = 0 \Rightarrow \frac{2}{3}x = \pi \Rightarrow x = \frac{3}{2}\pi$, dus $(\frac{3}{2}\pi, 4)$ of $(4\frac{1}{2}\pi, 4)$ etc

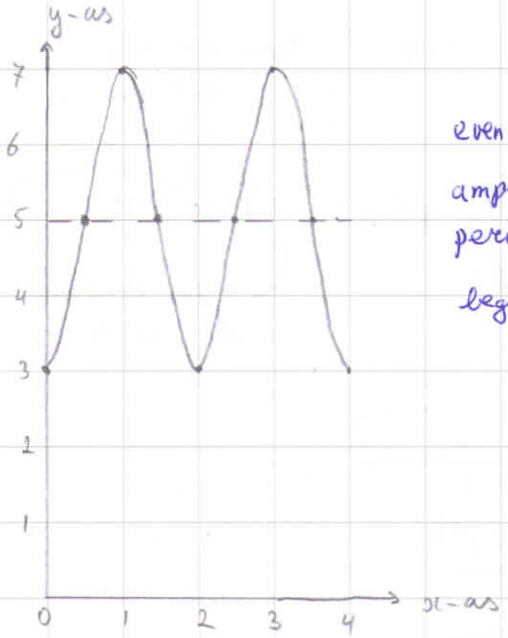
d) stel $2\pi(x+4) = 0 \Rightarrow x+4=0 \Rightarrow x = -4$, dus $(-4, 0)$ of $(-3, 0)$ etc

(21) a)



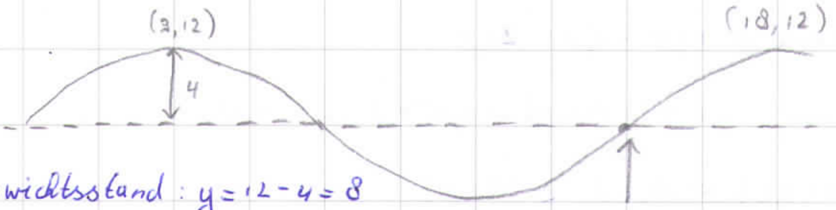
evenwichtsstand: $y = 3$
amplitude $= 2$
periode $= \frac{2\pi}{1/2} = 4\pi$
beginpunt : $(\frac{1}{2}\pi, 3)$

b)



evenwichtsstand: $y = 5$
amplitude $= 2$
periode $= \frac{2\pi}{\pi} = 2$
beginpunt : $(\frac{1}{2}, 5)$

(22) a)



evenwichtsstand : $y = 12 - 4 = 8$

$d = 3 + \frac{3}{4} \cdot 15 = 14,25$

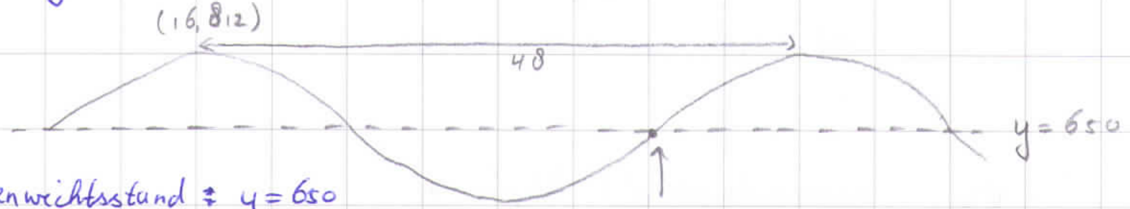
amplitude $= 4$

periode $= 18 - 3 = 15 \rightarrow c = \frac{2\pi}{15} = \frac{2}{15}\pi$

beginpunt : $(14,25 \,;\, 8)$

Dus $y = 8 + 4\sin\left(\frac{2}{15}\pi(x - 14,25)\right)$

b)



evenwichtsstand : $y = 650$

$d = 16 + \frac{3}{4} \cdot 48 = 52$

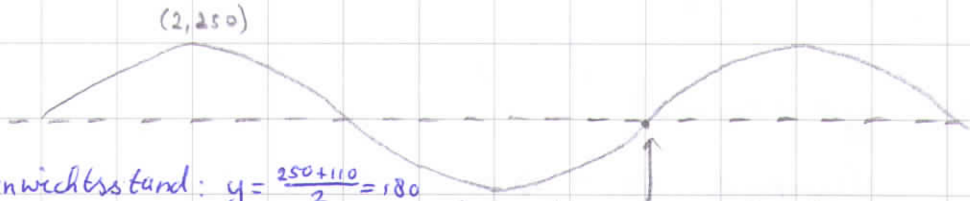
amplitude $= 812 - 650 = 162$

periode $= 48 \rightarrow c = \frac{2\pi}{48} = \frac{1}{24}\pi$

beginpunt bij : $(52, 650)$

Dus $y = 650 + 162\sin\left(\frac{1}{24}\pi(x - 52)\right)$

c)



evenwichtsstand : $y = \frac{250+110}{2} = 180$

(26, 110)

$d = 2 + \frac{3}{4} \cdot 48 = 38$

amplitude $= (250 - 110) : 2 = 70$

periode $= 2 \cdot (26 - 2) = 2 \cdot 24 = 48 \rightarrow c = \frac{2\pi}{48} = \frac{1}{24}\pi$

beginpunt bij : $(38, 70)$

Dus $y = 70 + 48\sin\left(\frac{1}{24}\pi(x - 38)\right)$

(23) $N = 30{,}8 + 6{,}3 \sin\left(\frac{2}{15}\pi(t-5{,}1)\right)$

a) $N_{max} = 30{,}8 + 6{,}3 = 37{,}1$ || $\text{periode} = \frac{2\pi}{2/15\,\pi} = 15$

$t_{max} = 5{,}1 + \frac{1}{4}\cdot\text{periode} = 5{,}1 + \frac{1}{4}\cdot 15 = 8{,}85$

b) $N_{min} = 30{,}8 - 6{,}3 = 24{,}5$

$t_{min} = 5{,}1 - \frac{1}{4}\cdot\text{periode} = 5{,}1 - \frac{1}{4}\cdot 15 = 1{,}35$

c) $N(7{,}2) \approx 35{,}65$ (trace)

d) $y_1 = 30{,}8 + 6{,}3\sin\left(\frac{2}{15}\pi(x-5{,}1)\right)$

$y_2 = 35$

optie intersect geeft: $x \approx 6{,}84 \lor x \approx 10{,}86$, $\Delta x \approx 4{,}02$

$\Delta t \approx 4{,}02$ uur ≈ 241 minuten

(24) evenwichtsstand $= \frac{4+-2}{2} = 1$

periode $= 8 \rightarrow c = \frac{2\pi}{8} = \frac{1}{4}\pi$

amplitude $= 4 - 1 = 3$

beginpunt bij $(6\frac{1}{2}, 1)$

Dus $y = 1 + 3\sin\left(\frac{1}{4}\pi(x - 6\frac{1}{2})\right)$

(25) $y = 2\sin(4x)$

beginpunt : $4x = 0 \Rightarrow x = 0$, dus $(0,0)$

periode $= \frac{2\pi}{4} = \frac{1}{2}\pi$

$x_{max} = 0 + \frac{1}{4}\cdot\text{periode} = 0 + \frac{1}{4}\cdot\frac{1}{2}\pi = \frac{1}{8}\pi$ (u_0 klopt)

opeenvolgende maxima zijn dus $\frac{1}{8}\pi, \frac{5}{8}\pi, \frac{9}{8}\pi, \frac{13}{8}\pi, \frac{17}{8}\pi, \ldots$

$u_n = \frac{1}{8}\pi + n\cdot\frac{1}{2}\pi$

$u_{20} = \frac{1}{8}\pi + 20\cdot\frac{1}{2}\pi = 10\frac{1}{8}\pi$