HET AMSTERDAMS LYCEUM

Naam: Hg : Rijen en Reeksen   Klas: V5   Cijfer:

Vak: Datum:

① a) $48, 51, 54, 57, 60, 63, \ldots$

$\underbrace{}_{+3} \underbrace{}_{+3} \underbrace{}_{+3}$ etc $\rightarrow$ constant verschil, dus RR

recursieve formule : $u_n = u_{n-1} + 3$ met $u_0 = 48$

directe formule : $u_n = 48 + 3n$

b) $20, 10, 5; 2,5; 1,25; 0,625; \ldots$

$\underbrace{}_{\cdot\frac{1}{2}} \underbrace{}_{\cdot\frac{1}{2}} \underbrace{}_{\cdot\frac{1}{2}}$ etc $\rightarrow$ constant quotiënt, dus MR

recursieve formule : $u_n = \frac{1}{2} \cdot u_{n-1}$ met $u_0 = 20$

directe formule : $u_n = 20 \cdot (\frac{1}{2})^n$

② 1 jan. 2006 : € 1000,-) +4% − € 100,-   ans = 1000 [ENTER]

1 jan. 2007 : € 940,-   ans · 1,04 − 100 [ENTER]

1 jan 2008 : € 877,60

⋮

1 jan 2018 : € 98,45

1 jan 2019 : € 2,39

1 jan 2020 : € −97,51 ← dus op 1 jan 2020 is saldo onvoldoende.

③ a) $u_n = n^3 - 3n + 1$

8e term, dus $u_7 = 7^3 - 3 \cdot 7 + 1 = 323$

b) $u_n = 5 + 2\sqrt{u_{n-1}}$ met $u_0 = 100$ ans = 100 [ENTER]

8e term, $u_7 \approx 11,90$ (via GR) $5 + 2\sqrt{ans}$ [ENTER]

④ $u_n = u_{n-1} + 6$ met $u_0 = 2$

u_n : $2, 8, 14, 20, 26, 32, \ldots$

$\underbrace{}_{+6} \underbrace{}_{+6} \underbrace{}_{+6}$ etc $\rightarrow$ constant verschil, dus RR

directe formule : $u_n = 2 + 6n$, dus $a = 6$.

⑤ a) $u_n = 2n+7$, $u_0 = 7$, $u_5 = 2\cdot 5 + 7 = 17$

$$\sum_{k=0}^{5} u_k \overset{RR}{=} \tfrac{1}{2}(5+1)(7+17) = 72$$

b) $v_n = n^2 + 3$

$$\sum_{k=0}^{4} v_k = v_0 + v_1 + v_2 + v_3 + v_4$$
$$= 3 + 4 + 7 + 12 + 19$$
$$= 45$$

c) $w_n = 2^n$

$$\sum_{k=0}^{4} w_k \overset{MR}{=} \frac{1\cdot(1-2^{4+1})}{1-2} = \frac{1-32}{-1} = 31$$

d) $z_n = 2u_{n-1} + n$ met $z_0 = 2$

$$\sum_{k=0}^{4} z_k = z_0 + z_1 + z_2 + z_3 + z_4$$
$$= 2 + 5 + 12 + 27 + 58$$
$$= 104$$

$z_0 = 2$
$z_1 = 2\cdot 2 + 1 = 5$
$z_2 = 2\cdot 5 + 2 = 12$
$z_3 = 2\cdot 12 + 3 = 27$
$z_4 = 2\cdot 27 + 4 = 58$

⑥ a) $17 + 21 + 25 + \dots + 149 =$

$$\sum_{k=0}^{33} u_k \overset{RR}{=} \tfrac{1}{2}(33+1)(17+149)$$
$$= 2822$$

($u_0 = 17$, steeds $+4 \to RR$)

$u_n = 17 + 4n$
$149 = 17 + 4n$
$132 = 4n$
$n = 33$
$u_{33} = 149$

b) $89 + 83 + 77 + \dots + 17 =$

$$\sum_{k=0}^{12} u_k \overset{RR}{=} \tfrac{1}{2}(12+1)(89+17)$$
$$= 689$$

$u_0 = 89$, steeds $-6 \to RR$
$u_n = 89 - 6n$
$17 = 89 - 6n$
$-72 = -6n$
$n = 12$
$u_{12} = 17$

⑦ a) $u_n = 7n + 1$

$$\sum_{k=0}^{n} u_k = \tfrac{1}{2}(n+1)(1+7n+1) = \tfrac{1}{2}(n+1)(2+7n) = \tfrac{1}{2}(7n^2 + 9n + 2)$$
$$= 3\tfrac{1}{2}n^2 + 4\tfrac{1}{2}n + 1$$

b) $\left.\begin{array}{l} y_1 = 3\tfrac{1}{2}x^2 + 4\tfrac{1}{2}x + 1 \\ y_2 = 1000 \end{array}\right\}$ optie table geeft: $x = \cancel{32}\ 16 \to y_2 = 969$
$x = \cancel{33}\ 17 \to y_2 = 1089$

Dus vanaf $\underline{n = 17}$

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(8) $12, 16, 20, 24, 28, \ldots$ steeds $+4 \rightarrow$ RR

$u_n = 12 + 4n$

$\sum_{k=0}^{n} u_k \overset{RR}{=} \frac{1}{2}(n+1)(12 + 12 + 4n) = \frac{1}{2}(n+1)(24+4n)$

$= (n+1)(12+2n)$

$= 2n^2 + 14n + 12$

(9) $\sum_{k=0}^{n} (3k) \overset{RR}{=} \frac{1}{2}(n+1)(0 + 3n) = \frac{1}{2}(n+1) \cdot 3n = 1\frac{1}{2}n^2 + 1\frac{1}{2}n$

(10) a) $\sum_{k=0}^{15} 100 \cdot 1{,}1^k \overset{MR}{=} \frac{100(1 - 1{,}1^{15+1})}{1 - 1{,}1} \approx 3594{,}97$

b) $v_n = 200 \cdot 0{,}98^n$

$\sum_{k=0}^{14} v_k \overset{MR}{=} \frac{200(1 - 0{,}98^{14+1})}{1 - 0{,}98} \approx 2614{,}31$

c) $w_n = 1{,}45 \cdot w_{n-1}$ met $w_0 = 50$

$w_n = 50 \cdot 1{,}45^n$

$\sum_{k=0}^{12} w_k = \frac{50(1 - 1{,}45^{12+1})}{1 - 1{,}45} \approx 13805{,}76$

(11) a) $u_n = 10000 \cdot 0{,}6^n$

$\sum_{k=0}^{n} u_k = \frac{10000(1 - 0{,}6^{n+1})}{1 - 0{,}6} = \frac{10000(1 - 0{,}6^n \cdot 0{,}6^1)}{0{,}4} = 25000(1 - 0{,}6 \cdot 0{,}6^n)$

$= 25000 - 25000 \cdot 0{,}6 \cdot 0{,}6^n$

$= 25000 - 15000 \cdot 0{,}6^n$

b) $\left. \begin{array}{l} y_1 = 25000 - 15000 \cdot 0{,}6^x \\ y_2 = 24999 \end{array} \right\}$ (optie intersect geeft $x \approx 18{,}8$

Dus $\underline{x = 19}$)

PS. eigenlijk optie table gebruiken ~~maar~~ maar is lastiger te interpreteren in dit geval.

$x = 18 \rightarrow y \approx 24998$

$x = 19 \rightarrow y \approx 24999$

(12) a) $u_n = 18 \cdot 0{,}4^n$

$$\sum_{k=0}^{n} u_k \overset{mR}{=} \frac{18(1-0{,}4^{n+1})}{1-0{,}4} = \frac{18(1-0{,}4^n \cdot 0{,}4^1)}{0{,}6} = 30(1-0{,}4 \cdot 0{,}4^n)$$
$$= 30 - 30 \cdot 0{,}4 \cdot 0{,}4^n$$
$$= 30 - 12 \cdot 0{,}4^n$$

b) $\left.\begin{array}{l} y_1 = 30 - 12 \cdot 0{,}4^x \\ y_2 = 29{,}99 \end{array}\right\}$ optie table geeft: $x = 7 \rightarrow y = 29{,}98$

$x = 8 \rightarrow y = 29{,}992$

Dus vanaf $n = 8$.

(13) $u_n = u_{n-2} + 3u_{n-1}$ met $u_0 = 2$ en $u_1 = 5$

$u_2 = u_0 + 3u_1$ (overal $n=2$ invullen)

$u_2 = 2 + 3 \cdot 5 = 17$

$u_3 = 5 + 3 \cdot 17 = 56$

$u_4 = 17 + 3 \cdot 56 = 185$

$u_5 = 56 + 3 \cdot 185 = 611$

$u_6 = 185 + 3 \cdot 611 = 2018$

$u_7 = 611 + 3 \cdot 2018 = 6665$

$u_8 = 2018 + 3 \cdot 6665 = 22013$

(14) $u_{n+2} = u_n^2 + u_{n+1}$ met $u_0 = 2$ en $u_1 = 5$

$u_2 = u_0^2 + u_1$ (overal $n=0$ invullen)

$u_2 = 2^2 + 5 = 9$

$u_3 = 5^2 + 9 = 34$

$u_4 = 9^2 + 34 = 115$

$u_5 = 34^2 + 115 = 1271$

$u_6 = 115^2 + 1271 = 14496$

$u_7 = 1271^2 + 14496 = 1629937$

$u_8 = 14496^2 + 1629937 = 211763953$

(15) a) $u_n = 11{,}3 \text{ mjd} \cdot 1{,}074^n$

b) $$\sum_{k=0}^{12} u_k \overset{mR}{=} \frac{11{,}3 \text{ mjd}(1-1{,}074^{12+1})}{1-1{,}074} \approx 233{,}58 \text{ mjd dollar}$$

(16)

$u_0 = 18$) $+5,2$

$u_1 = 23,2$) $+5,2 \cdot 0,8$

$u_2 = 27,36$

$\vdots$

$u_n = 5,2 \cdot 0,8^n$

a) groei in 8^e week: $u_7 = 5,2 \cdot 0,8^7 \approx 1,1$ cm

b) $\sum_{k=0}^{7} u_k = \dfrac{5,2\,(1-0,8^{7+1})}{1-0,8} \approx 21,6$ cm

c) $\sum_{k=0}^{9} u_k = \dfrac{5,2\,(1-0,8^{9+1})}{1-0,8} \approx 23,2$ cm

Dus hoogte is dan $18 + 23,2 = 41,2$ cm

(17)

$\underbrace{4,9 \;;\; 14,7}_{+9,8} \;;\; 24,5$ etc $\rightarrow$ constant verschil van $+9,8 \rightarrow$ RR

$14,7 \;;\; 24,5$: $+9,8$

$u_n = 4,9 + 9,8n$

$\sum_{k=0}^{n} u_k \overset{RR}{=} \frac{1}{2}(n+1)(4,9 + 4,9 + 9,8n)$

$= \frac{1}{2}(n+1)(9,8 + 9,8n)$

$= (n+1)(4,9 + 4,9n)$

$= 4,9n^2 + 9,8n + 4,9$ Dus $a = 4,9$, $b = 9,8$, $c = 4,9$

(18) a) $\sum_{k=2}^{9}(5k+6) \overset{RR}{=} \frac{1}{2}(9+1-2)(16+51)$

wegens ontbreken $k=0$ en $k=1$

$= \frac{1}{2} \cdot 8 \cdot 67$

$= 268$

$u_2 = 5 \cdot 2 + 6 = 16$

$u_9 = 5 \cdot 9 + 6 = 51$

u_2 is eerste term.

b) $\sum_{k=2}^{9}(3 \cdot 2^k) \overset{MR}{=} \dfrac{12 \cdot (1-2^{9+1-2})}{1-2}$

$= \dfrac{12 \cdot (1-2^8)}{-1}$

$= 3060$